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Active vibration control of smart building frames by feedback controllers

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Abstract

In this paper the active vibration control of a four-story shear frame instrumented with piezoelectric actuators is presented. The piezoelectric actuators are hosted on the columns in two manners and the produced controlling forces by actuators are considered in the equation of motion. The smart structure modeling and control design is carried out using MATLAB software in state space form. Subsequent details of designing feedback control are addressed. Pole Placement Controller (PPC) and Linear Quadratic Regulator (LQR) are applied in control algorithms and their performances are discussed. The results of both methods are compared which show excellent agreement, demonstrating that piezoelectric based structural control is a proper approach for optimized smart structure design. Results also show that the location of the piezoelectric actuators significantly influences the efficiency of the control system.

Keywords: Active vibration control; Piezoelectric actuators; Pole placement controller; Linear quadratic regulator.

1. Introduction

The suppression of vibration is an important problem in many engineering applications. Various passive vibration control techniques have been utilized to diminish the structural vibrations. Examples include using tuned-mass-dampers, base isolations, friction dampers, viscous dampers and so on in civil structures [1]. However, the functionality of passive systems cannot be guaranteed against the vibration sources like earthquakes which content different dominant frequencies. Also passive devices are usually weighty, while in many applications it is desirable to keep the weight as low as possible.
All structural systems reveal vibration response due to external excitation. In order to modify the system response in a desired way, it can be added secondary inputs to the structure. This is performed in active vibration control, which has the ability to actuate the system in a

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controlled manner. Modeling of active structures has attracted a lot of researchers in the past decades [2, 3].

An active structure or smart structure includes three main parts, which are sensors, controllers and actuators. Sensor measures the structural response and its outputs are fed to a controller which controls inputs for actuating the structure, in order to mitigate the structural vibration [2]. It is clear that transforming electrical energy to mechanical energy and vice versa is needed in a control system.

Piezoelectricity is a coupling between a material's mechanical and electrical behaviors. When a piezoelectric material is strained, an electric charge is generated. Conversely, when a piezoelectric material is subjected to a voltage drop, it mechanically deforms. Therefore, they can be used as both actuators and sensors in smart structures. In order to applying these devices in smart structures many researches have been performed [4-7].

Song et al. [8] considered the active vibration control of a space truss using a Lead Zirconate Titanate (PZT) stack actuator. Trindade et al. [9] and Xu et al. [10] presented results for active control of beams using piezoelectric patch actuators, showing again the potential of piezoelectric actuators in vibration control.

Some of the investigators examined the use of piezoelectric actuators in vibration control of frame type structures. Kamada et al. [11] developed a four-story building model, which was controlled by piezoelectric actuators in different manners. They succeeded to reduce the floors accelerations up to 70 %. Sethi et al. [12] experimented a three-story model frame, equipped by a piezoelectric patch sensor and actuator. They successfully implemented pole placement modal controller to control all of the structural modes. Kwak et al. [13] also discussed the vibrating behavior of a two-story smart frame using finite element modeling and positive position controller.

It is noteworthy that the various types of controllers exist for response control of structures. For building structures, modal controllers such as model matching, positive position, H_∞ and pole placement may be more appropriate to control the first dominant modes [11, 12]. Beside them, the optimal controllers as LQR have been developed and used in practical implementations [14-16].

The primary aim of this study is to investigate the potential of the piezoceramic stack actuator in feedback active control of seismic vibration. These actuators are suitable specifically for use in structures with stiffeners added. A mathematical model is developed to describe the response of frame to excitation caused by the real earthquake record. The solutions of the classical equations of motion for frame structures and the control of the structural responses using control schemes and stack actuator is presented. Both LQR and pole placement controllers are applied and their performances are compared.

2. Structure modeling

2.1. Piezoelectric actuators

Piezoelectric ceramics are most popular and most powerful piezoelectric materials which is available in different shapes and dimensions. Among them, PZT actuators can generate a strain up to 500μ strain and a blocked force up to 100 KN proportional to the applied electric voltage.

This feature has made them to become attractive in active vibration control of structures.

2.2. Frame structure

In order to design and analyze an active structure, it is important to have a good understanding of the vibration characteristics of the system under consideration. An important task is therefore to construct some kind of model for the actual system. This can be performed in various ways. The most common approach is to utilize physical relations to obtain a mathematical model that describes the system.

Finite element modeling is a popular technique for spatial discretization of distributed parameter systems (such as beams and plates) which yields spatially discrete models of very high order.

For the frame structures, dynamic structural analysis is carried out usually based on lumped parameter model. In this system, it is assumed that the structural mass and stiffness have been concentrated in floors and columns, respectively.

Damping is also assumed to be a linear combination of mass and stiffness matrices [17].

Therefore, the equation of motion of a linear building frame structure with n degrees of freedom and equipped with piezoelectric actuators, subjected to a seismic excitation is presented as:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F_m\}+\{F_c\} \tag{1}$$

in which M, C and K are mass matrix, damping matrix and stiffness matrix, respectively. q, F_m and F_c represent the floor displacement, external excitation and control force which is exerted by piezoelectric actuators, respectively.

3. Design of controllers

In this study, the linear quadratic regulator and pole placement controller are considered as active vibration controllers for the frame structure. The dynamic equation of the system in the state space representation given by:

$$\begin{cases} \dot{x} = Ax + B_c u + B_p \\ y = Cx \end{cases} \tag{2}$$

with state vector x, input vector u and output vector y, A, B_c and C represent system, control and state output matrices, respectively, and B_p is external excitation vector. These have the following expressions:

$$A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad ; \quad x=\begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$$

$$B_p=\begin{bmatrix} 0 \\ M^{-1}F_m \end{bmatrix} \qquad B_c=\begin{bmatrix} 0 \\ M^{-1}F_c \end{bmatrix}$$

The optimal control is obtained through full state-feedback with a control law defined as follows:

$$u=-Kx \tag{3}$$

In other words, the control voltage u is proportional to the state vector x. K expresses the control gain in the above equation.

Consequently:

$$\dot{x}=(A-B_c K)x+B_p \tag{4}$$

3.1. Linear Quadratic Regulator

Linear quadratic regulator optimal control theory is used to determine the control gains. The feedback control system is designed to minimize a cost function or a performance index, which is proportional to the required measure of the system’s response. The cost function used in this case is given by [2]:

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{5}$$

where Q is semi positive definite ($Q \geq 0$) and R is strictly positive definite ($R > 0$). J has two contributions, one from the states x and the other one from the control u.
The gain matrix, K, can be obtained by solution of Riccati equation given by [2]:

$$A^T P + PA - PB_c R^{-1} B_c{}^T P + Q = 0 \tag{6}$$

$$K = R^{-1} B_c^T P \tag{7}$$

where P is an auxiliary matrix.

3.2.Pole Placement controller

In this technique the control effort is performed to vibration suppression of n different modes, using n independent gain matrix. For this purpose, the gain matrix is obtained using Ackermann's method, after choosing desired closed loop poles [2].

$$K = B_c^T S^{-1} \alpha_c(A) \tag{8}$$

in which,

$$S = [B_c \;\; AB_c \;\; A^2 B_c \;\; \dots \;\; A^{n-1} B_c] \tag{9}$$

$$\alpha_c(A) = A^n + \alpha_{n-1} A^{n-1} + \cdots + \alpha_1 A + \alpha_0 I \tag{10}$$

where S is the controllability matrix. For appropriate placement of closed loop poles, this matrix determinant must not be zero. This characteristic is known as controllability.

4. Numerical study

In order to assess the ability of piezoelectric actuator in active vibration control, a planar four-story building frame, with one degree-of-freedom per floor, resulting in a total of four degrees of freedom is considered. This structure has a weight of about 2000 kg and a height of about 4 m. It is composed of four layers, and each layer weighs about 500 kg [11]. (Figure 1)

The piezoelectric actuators placement is considered in two cases which are described as follows. The specifications of these devices are listed in table 1.

Table 1. Specifications of piezoelectric actuators [11].

Manufacturer	Tokin Corporation
Type	NLA-25×25×36
Dimension (mm)	25×25×36
Rated Voltage (V)	100
Provided Force (kN/100 V)	19.8

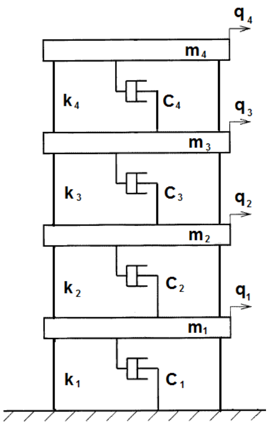


Figure 1. Structural Model.

4.1. Actuators placement in case#1

In this case, which have been introduced by *Kamada et. al.*[11], piezoelectric stack actuators are placed at the bottom of first story columns in both sides. If the induced voltage is applied in reverse phase to these actuators, the concentrated bending moments M_a are produced at the bottom of columns. By applying the basic structural calculations, the equivalent shear force at each story level is determined as follows (Figure 2):

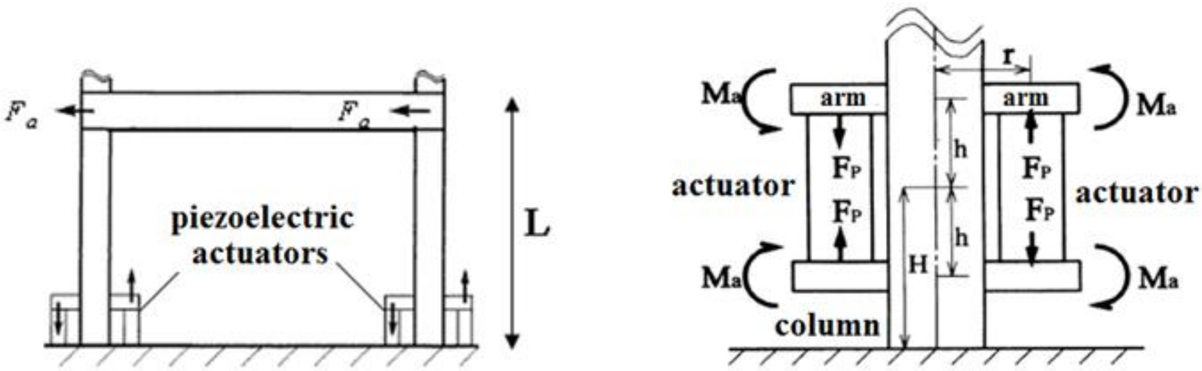


Figure 2. Piezoelectric actuators placement in case#1.

$$M_a = F_p r \tag{11}$$

$$F_a = \frac{24hr}{L^3}(L-2H)F_p \tag{12}$$

where L, F_p and F_a are the columns height, actuators produced force per unite voltage and equivalent shear force produced by actuators in first floor level, respectively. Other parameters have been shown in Figure 2.

4.2. Actuators placement in case#2

In case#2, a new method for seating the actuators is introduced. In this case, two actuators are placed between the two angle profiles, which are installed at the bottom of the first story columns, so that the actuators' axis is perpendicular to the columns' axis (see Figure 3). Similar to the case#1, the equivalent shear force acting on the first floor is obtained using the basic structural analysis:

$$F_a = \left(\frac{2r}{L}\right)^3 F_p \tag{13}$$

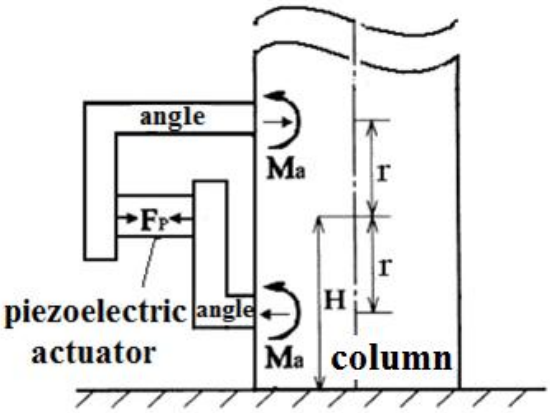


Figure 3. Piezoelectric actuators placement in case#2.

In both propounded methods, the moment arm r plays an important role in control force magnitude.

According to equation (12), it is manifestly clear that by placing the actuators at the nearest distance from the base, the best result can be obtained.

According to equations (12) and (13), Figure 4 shows that for moment arm less than 25 cm, the control force in two cases are the same. While for greater moment arm, case#2 is more effective.

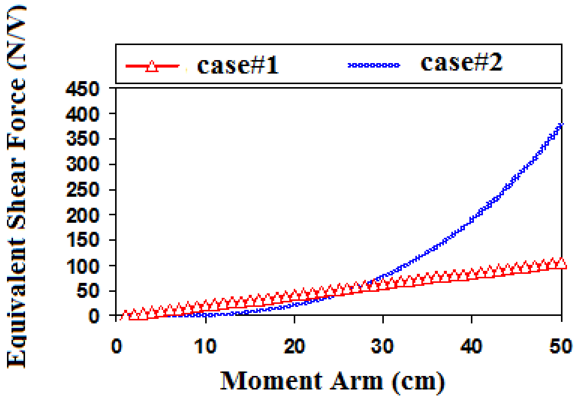


Figure 4. Comparison of control force in two cases with regard to change of moment arm.

So with respect to the required control force and practical limitations, the moment arm is chosen as 35 cm.

4.3. Controllers design

Using try and error approach, the weighing matrices Q and R are chosen as follows:

$$\begin{aligned} &R = I \ , \\ &Q = W \times [Q_1, Q_2] \\ &Q_1 = I \ , Q_2 = 0.001 Q_1 \\ &W = 1.7 \times 10^{10} \quad for\ case\#1 \\ &W = 5.7 \times 10^{10} \quad for\ case\#2 \end{aligned} \tag{14}$$

So for the LQR controller, the gain matrix K in case#1 consists of:

K = [1.47E+03 -1.27E+03 -9.03E+01 2.44E+02 1.42E+03 7.99E+02 5.09E+02 3.93E+02]

And in case#2:

K = [1.32E+04 -1.33E+04 -5.10E+02 1.86E+03 4.33E+03 2.24E+03 1.44E+03 1.14E+03]

In this research, a pole placement controller is also designed to control the all modes of the structure.

For this, first using the transfer function of the system, systems' poles are determined and then, by observing the controllability conditions and keeping the control force in allowable range, closed loops' desired poles are assigned (Table 2).

Table 2. Existing and desired pole locations.

Mode	Existing poles	Desired poles (case#1)	Desired poles (case#2)
1	-1.52±152.11i	-1.51±873.56i	-1.51±873.56i
2	-4.38±437.97i	-8.96±695.98i	-8.96±695.98i
3	-8.76±670.98i	-14.38±448.97i	-14.38±448.97i
4	-12.6±823.06i	-11.52±153.11i	-39.52±153.11i

For the PPC the gain matrix *K* which is obtained using Ackermann method, in case#1 consists of:

K = [6.54E+07 -5.51E+07 2.89E+07 -1.04E+07 2.27E+04 3.82E+04 3.45E+04 2.34E+04]

And in case#2:

K = [6.79E+07 -4.98E+07 3.37E+07 -6.35E+06 6.87E+04 1.56E+05 1.83E+05 1.99E+05]

The frame structure is excited by the Imperial Valley earthquake record and structural responses are controlled using two control schemes with two actuators' placement cases.

5. Results and discussions

A computer code has been developed in MATLAB environment for the dynamic vibration control of the smart frame using LQR and PPC in two placement cases. The uncontrolled and controlled story displacement, acceleration, drift and shear force is presented in Figure 5 for LQR and Pole Placement (PP) controllers of case#1. It is observed that two controllers have successfully reduced the structural responses. However, PP controller is a bit more efficacious than LQR controller.

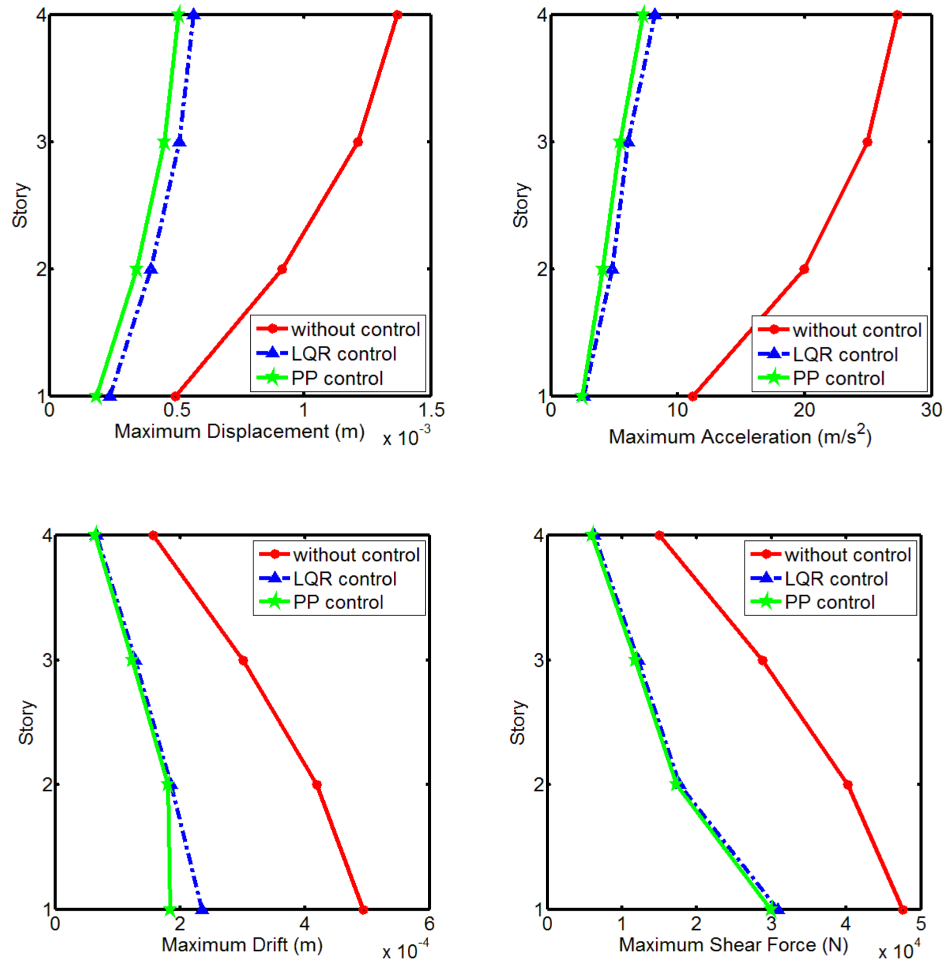
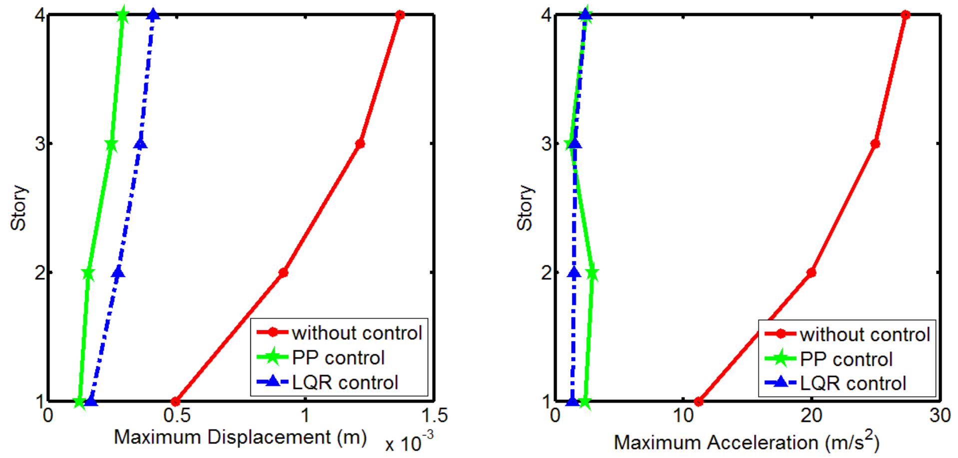


Figure 5. Comparison of LQR and PP controlled structural response in case#1.

For case#2, uncontrolled and controlled structural responses have been presented in Figure 6. Results indicate that piezoelectric actuators are capable to mitigate the vibration of frame structure. Also it is perceived that the floors accelerations are subjected to more reductions comparing to other responses.



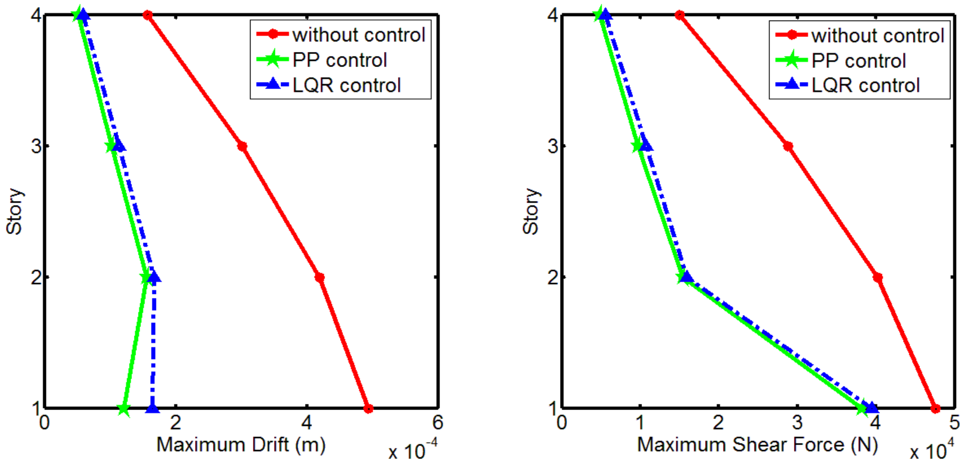


Figure 6. Comparison of LQR and PP controlled structural response in case#2.

Figure 7 shows the LQR based controlled response of structure. Results indicate that the LQR scheme has a noticeable effect on structural response reduction such that the stories maximum responses including displacements, accelerations, drifts and shear forces are reduced by 59 % , 77 %, 58 % and 58 % for case#1, respectively, while this reduction is 63 %, 86 %, 60 % and 60 % for case#2, respectively.

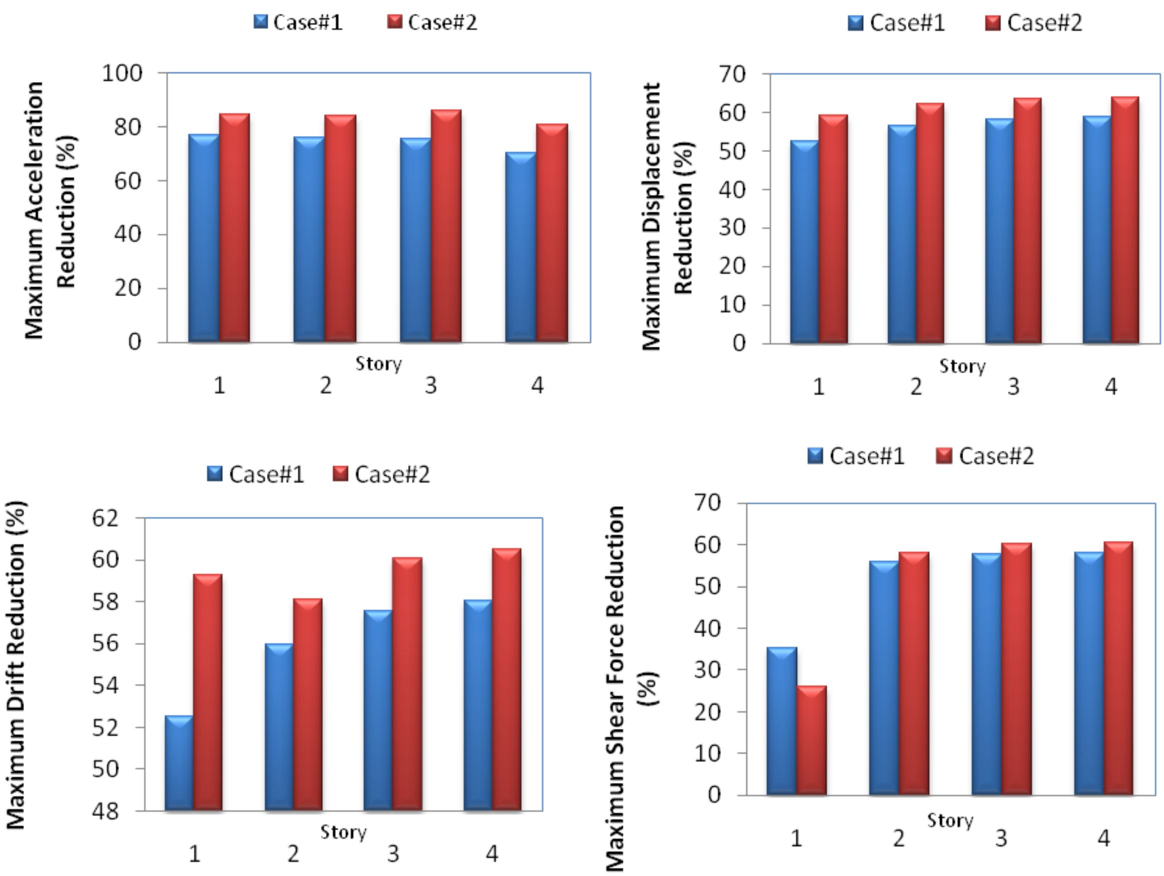


Figure 7. Comparison of LQR based structural response for two placement cases.

As shown in Figure 8, which is related to the PP based structural response control, it's perceived that for both cases the maximum responses including displacement, acceleration, drift and shear force are reduced approximately by 63 %, 80 %, 63 % and 60 %, respectively. Also it is observed that while the maximum acceleration and drift reduction occurs in the lower floors, the maximum story displacement and shear force reduction can be observed in the top floor. Like previous observations in PP controller, the efficiency of case#2 in response reduction is observed.
Generally, it can be concluded that utilizing actuators placement of case#2 leads to a more appropriate control performances.

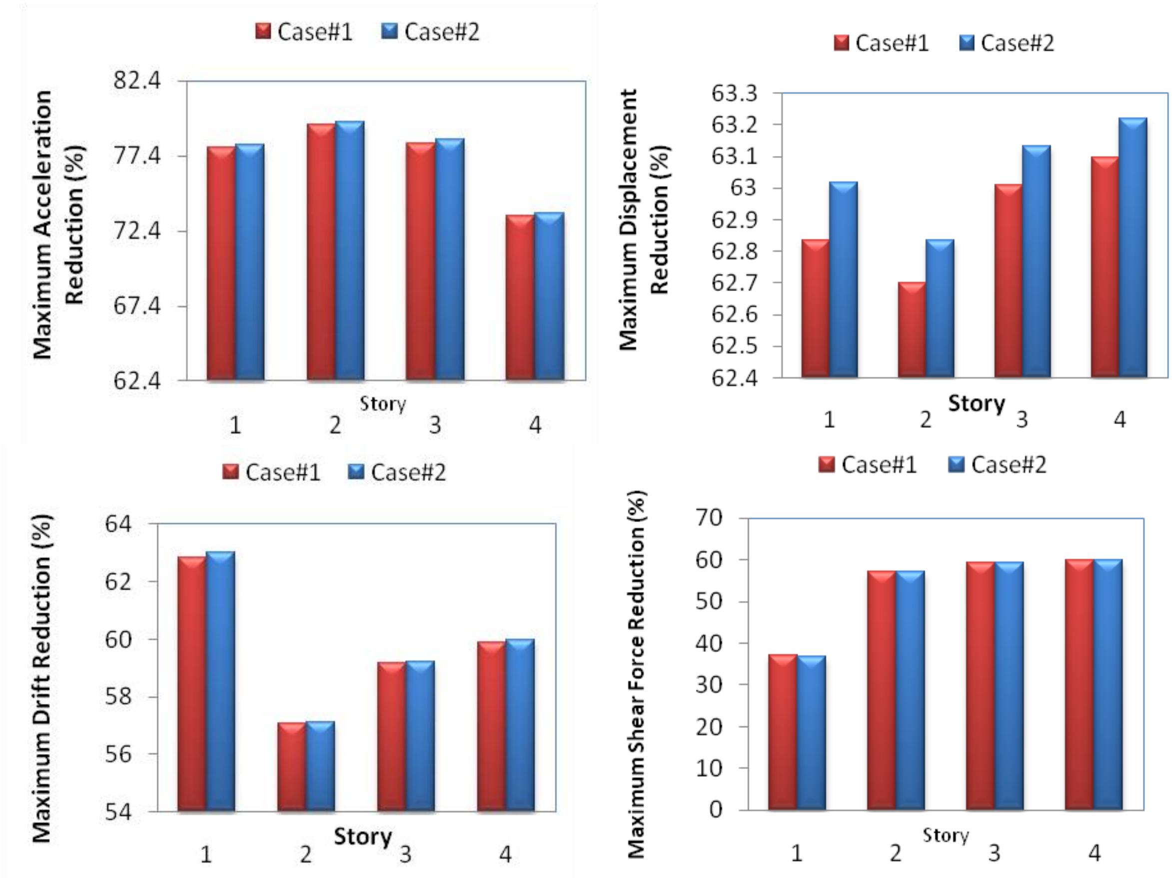


Figure 8. Comparison of PP based structural responses for two placement cases.

6. Conclusion

Simulation of a four-story model frame equipped with piezoelectric stack actuators was performed in this study. The uncontrolled and controlled structural responses due to the seismic excitation were considered corresponding to two different cases of actuators placement and two different control algorithms.
It was observed that the presented schemes reduce the structural response considerably. However by implementing case#2 in actuators' placement and using PP controller, the best results can be achieved.

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