

A comparative study on pile group and piled raft foundations (PRF) behavior under seismic loading

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Abstract

Study on the seismic behavior of piled rafts and pile groups while the same amount of construction material and excavation is used in their construction, are the main objective of this research. The process where the raft interaction with soil can affect the seismic response and stress distribution is also discussed in the current study. By means, ABAQUS software was applied for the finite element modeling. Firstly, for calibration and verification of the procedure of modeling with two other experimental studies, results were compared by the analysis of models under single frequency sinusoidal dynamic loads. Moreover, analyses of the same models were applied under the acceleration time history of the El-Centro earthquake. Results indicate that the participation of the raft interaction with soil under seismic loading, well improves the seismic response and behavior of a piled foundation systems i.e. internal moments and shear forces as well as deflection. Reductions of maximum acceleration response, horizontal displacement and bending moment in the piled raft system, are some important gained results by this study. In addition, the more uniform stress distribution in soil deposit and prevention of stress concentration leads to the reduction of the piled rafts settlement during earthquakes.

Keywords: Piled raft foundations; Finite element modeling; Earthquake loading; Seismic response.

1. Introduction

All engineered construction resting on the earth must be carried by some kind of interfacing element called a foundation. The term of foundation describes a structural element that connects a structure to the ground [1]. Rafts are realized to be a reasonable foundation system for medium sized buildings, silos and Non-river bridges basically when the structural loads are so high or the soil condition regarding its stiffness and strength is poor. If a shallow foundation is not adequate, it is common in foundation engineering to design a fully piled foundation in which the entire loads are transferred to the subsoil by piles [2].

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Recently, by the improvement of accuracy in geotechnical engineering the beneficial utilization of construction materials should be considered in foundation design. In traditional methods of piled foundation design, because of the occurrence of large settlements under the pile cap resulted in the separation of the raft and soil, therefore in the calculations of bearing capacity of foundations only the piles were considered and no emphasis was made on the raft as a load sharing element.

To date, according to the advanced numerical analysis, one can consider the interaction between a raft and the soil in foundation design. Among such design improvements, it is common for a raft to be part of the foundation system. In recent years, there has been an increasing recognition that the use of piles to reduce rafts total and differential settlements, can lead to considerable economy without compromising the safety and performance of the foundation. Such a foundation makes use of both the raft and the piles, and is referred to as a piled raft foundation (PRF) [3].

The piled raft foundation system is a type of composite foundation which involves the contribution of piles, raft, and soil to transmit heavy loads of the superstructure to the ground. In the design of piled rafts the load sharing between the piles and raft is taken into account. The use of PRF is an effective way of minimizing both total and differential settlements, improving the bearing capacity of a shallow foundation, and effectively reducing the internal stress levels and bending moments within a pile [4].

Moreover, utilizing foundations that share the contributions of both raft and piles in tall and heavy buildings, or otherwise when extraordinary environmental loadings such as earthquake, wind and wave loads exist or more generally, when the condition of soft and loose subsoil is observed, piled raft foundations seem to be a quite reasonable implementation.

The vertical load bearing mechanism has been extensively investigated by a number of researchers by applying the elasticity theory [5,6] and the finite element method [7,8]. On these results, piled raft foundations are becoming popular in practical use and with this type of foundation design becoming a common procedure, the need of study on its seismic behavior seems necessary. The study on the load bearing mechanism under horizontal loading or during earthquakes, however, is very limited in the literature. Since the behavior of a piled raft foundation during earthquakes is considered fairly complex due to dynamic interaction among a raft, piles and a soil, the design procedure should include the effect of this mechanism in an appropriate manner. The objective of this study is to generalize this behavior, i.e. the static behavior, to dynamic conditions, under earthquake loading and by three dimensional finite element modeling in custom engineering scales.

2. Overview on modeling process

The main objective of finite element modeling via ABAQUS is the investigation of pile-soil interactions under seismic or earthquake loading. In this study effective parameters in the modeling are checked in order to reach a better procedure of analyzing the soil-pile interactions. The main points of this model are as follows:

2.1. Semi infinite medium modeling

One of the most important parameters in the modeling is the seismic behavior of soil and the manner of the propagation of seismic waves in a semi infinite medium. By utilizing usual boundary conditions in the static modeling such as constraints, the waves are reflected inside the model and create unacceptable results (box effect). Therefore, in order to overcome such

shortcomings and to be able to model the radiational damping one can use the following options in the software:

- Utilizing dashpot springs (Kelvin elements) and equivalent horizontal effective stresses, according to Rayleigh's method this semi infinite medium is equivalent to dampers and stiffeners. In this method, the soil is assumed to be at larger distances in the medium. Figure 1 shows the soil medium and boundary conditions of the piled raft model (indicating that the raft is in contact with soil). The horizontal boundary conditions are chosen to be dashpot springs (Kelvin elements) along with equivalent horizontal effective stresses [9,10]. The bottom boundary condition of the model is a constraint inhibiting in y-direction where the height of the model is 16 m with the width of 28 m. The dynamic equation for this system is [11]:

$$m\ddot{x} + c\dot{x} + kx = 0 \quad Or, \quad \ddot{x} + 2\xi\omega\dot{x} + \omega^2x = 0 \tag{1}$$

where, $\omega^2 = \frac{k}{m}$ and $\xi = \frac{c}{2\sqrt{mk}}$, m is the mass of element, \ddot{x} , \dot{x} and x are acceleration, velocity and displacement, respectively. Also, c is the damping coefficient and k is the stiffness factor.

The damping ratio for the dashpot boundary conditions is calculated by the following [11]:

$$C_d = \rho \cdot V_s \cdot A \tag{2}$$

where ρ is the soil density, V_s is the velocity of shear wave and A is the equivalent area of the spring.

The stable time increment for the spring-dashpot system is:

$$\Delta t_{stable} = \frac{2}{\omega} (\sqrt{1 + \xi^2} - \xi) \tag{3}$$

As the dashpot coefficient c is increased, the stable time increment, Δt_{stable} , will be reduced.

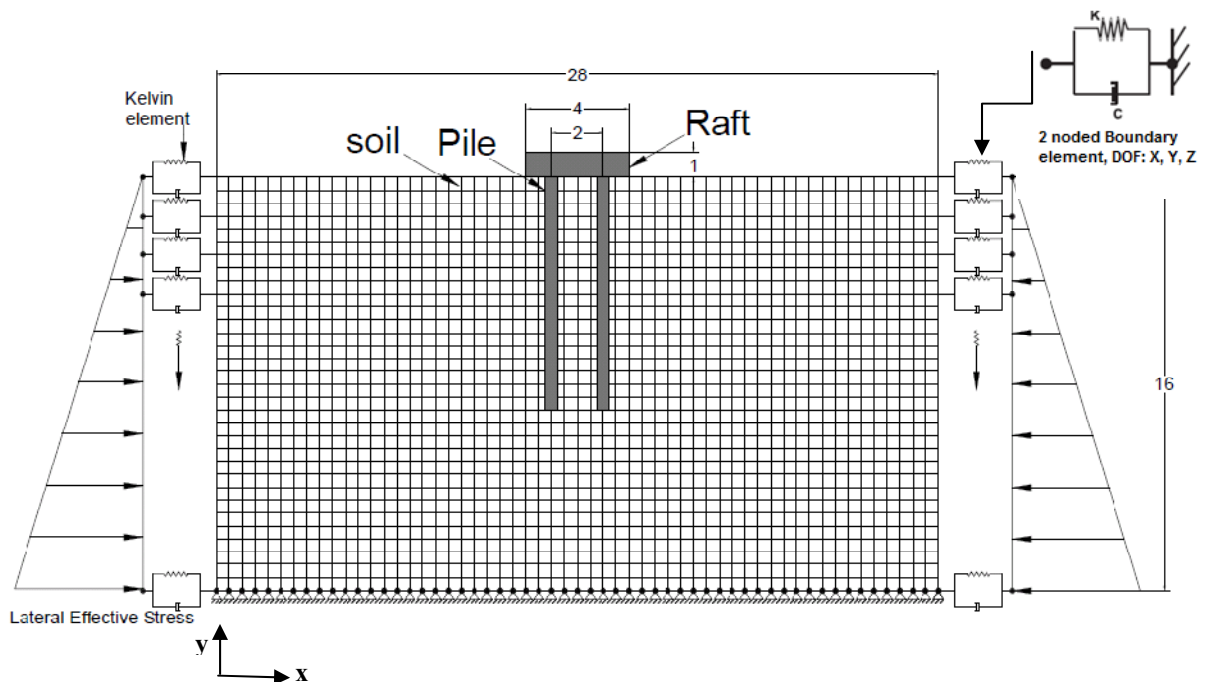


Figure 1. Schematic illustration of finite element modeling of piled raft.

The stiffness of the springs (K_s) is a function of the Young's modulus of the soil. Moreover, lateral soil pressure should be applied along the depth of soil according to $K_0 \cdot \gamma \cdot h$ distribution. In this case an appropriate proportion should be considered between mesh dimensions and the distance between springs. If the intervals between dashpot springs are too much, C_d and K_s reach large values and if a fine mesh is used, distortion of elements occurs because there would be some elements between the dashpots for which, no horizontal restraint exists and hence, they tend to deform unlike others connected to a dashpot. This dashpot makes the boundary act as an absorbent boundary due to absorbance of the received energy. It is different from absorbent boundaries assumed in other similar FEM codes which replace the horizontal displacement boundary with a horizontal stress boundary of opposite site. Therefore, the arrangement of dashpot springs is dependent on mesh dimensions and Young's modulus of the soil.

In the development of models in this study, the entire model is chosen to be adequately large with dashpot springs, so that the boundary conditions do not affect the analysis results.

2.2. Pile-soil interaction

Two popular interactions were used in the model and the results were compared to each other. First, the tie method, in which, pile elements are in full contact with the soil elements and there is no relative displacement between them. The results of this approach indicate good agreement with other numerical models, whereas there are some differences with experimental investigations [12,13]. These variations are due to separation between piles and soil in experimental studies. Therefore, another kind of contact was applied in both normal and tangential directions, whereas in the normal direction hard contact was utilized and frictional behavior was chosen for the tangential direction. According to experimental studies the frictional coefficient was set to 0.423 [12]. In addition, to prevent the separation between the raft and soil a cohesive contact was implemented in the model, which the maximum bearable tensional stress was limited to 10 kPa. For instance, Figure 2 illustrates the horizontal displacement between the pile and soil in pile edges in one arbitrary point at mid-pile length. According to this figure, separation between pile and soil can be observed in the peak points.

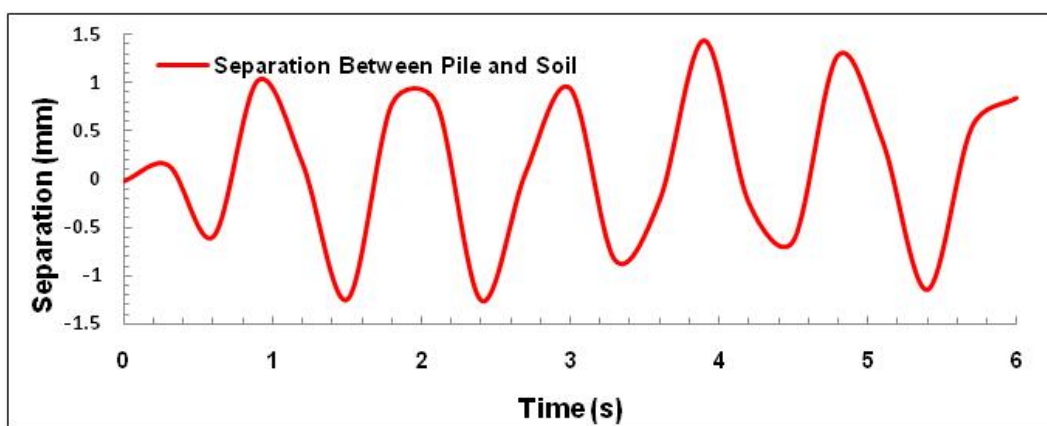


Figure 2. Horizontal displacement between pile and soil at one arbitrary point on pile edge.

2.3. Material damping

In order to model this type of damping for the soil α and β coefficients of Rayleigh formulation are used. In this case by utilizing an elastic analysis in frequency domain two

natural frequencies which approximately have 90 % of mass participation was chosen and by the following formulations the values of α and β were obtained [11] which is also called the Rayleigh damping defined in many texts:

$$\alpha\omega_1 + \frac{\beta}{\omega_1} = \xi = 5\% \quad (4)$$

$$\alpha\omega_2 + \frac{\beta}{\omega_2} = \xi = 5\% \quad (5)$$

2.4. Nonlinear behavior of soil

On the basis of the two experimental papers [13,14] a nonlinear elastic-perfect plastic model with a Mohr-Coulomb yield criterion was adopted. Therefore, input parameters were as follows;

Friction angle (φ) = 31°, dilation angle (ψ) = 1°, soil cohesion (c) = 10 kPa, meridional eccentricity = 0.1.

2.5. Elements and meshing

According to the formulation of tetrahedron elements the strain is constant in the element. Therefore, by using this kind of elements very fine mesh is necessary. But in order to reduce the analysis execution time hexagonal elements were used, to increase the size of elements and to obtain better accuracy in the analysis because of linear variation of strain along the elements.

In the pile meshing process because of the dimensions of the pile (0.5m×0.5m) using the mesh dimension of 0.5m×0.5m×0.5m can be sufficient. Additionally, the defaults of the program are set on C3D8R elements (cubic three dimensional-reduced) for the analysis. These elements have one integration point and can cause an interlocking phenomenon. This problem occurs when the integration point gets placed on the neutral axis of the bending member, and as a result, the bending moments calculated lack enough accuracy needed in this study. Therefore, in order to overcome such problems, the mesh dimensions were decreased and set to 0.25m×0.25m×0.25m with regular cubic elements (C3D8) having eight integration points.

2.6. Analysis process

In order to reduce the analysis execution time, explicit analysis procedure was chosen. By using this method more necessity is seen in checking the results such as:

- The explicit procedure integrates through time by using many small time increments. The central-difference operator is conditionally stable, and the stability limit for the operator (with damping) is given in terms of the highest frequency of the system as [11]:

$$\Delta t \leq \frac{2}{\omega_{max}} (\sqrt{1 + \xi_{max}^2} - \xi_{max}) \quad (6)$$

where ξ_{max} is the fraction of critical damping in the mode with the highest frequency.

- In a quasi-static analysis it is expedient to reduce the computational cost by either speeding up the simulation or by scaling the mass. In either case the kinetic energy was monitored to ensure that the ratio of kinetic energy to internal energy does not get too large, typically less than 10%.

3. Verification of models

In order to verify the modeling procedure via ABAQUS, three different models were analyzed and calibrated. The first model was a simple raft foundation which was chosen from experimental centrifugal test studies [14]. The second and third models were obtained from another experimental centrifugal test report [12], which had tested a free standing pile group and also a piled raft foundation, comparing the results by obtaining displacements, accelerations, and bending moments. These models are thoroughly described in the following:

3.1. Simple raft foundation

In this model a simple concrete square raft with 6.24 m width and 1.47 m thickness was set up on a 22.5×13.5 m soil (dry Toyoura sand) with depth of 12 m. Soil properties are given in Table 1.

Table 1. Toyoura sand properties applied in this study [15].

Young's modulus (kN/m^2)	4×10^4
Soil unit weight (kN/m^3)	16.3
Poisson's ratio	0.3
Friction angle (φ)	31°
Dilation angle (ψ)	1°
Soil cohesion (c) (kPa)	10

In the centrifugal test [14] an artificial earthquake wave (developed based on the energy content of an earthquake) with the amplitude of 1.8 m/s^2 in the prototype scale was applied to the model. However, in the current study a sinusoidal wave with the amplitude of 1.8 m/s^2 with the frequency of 1 Hz which is very similar to the centrifugal test were applied to the model. Results indicate that the horizontal displacement on the soil was 5.64 cm whereas the displacement on top of the raft was calculated as 6.18 cm. Also, the accelerations on top of the raft were measured to be 3.3 m/s^2 with the frequency of 1.28 Hz, whereas the accelerations seen on the soil were 3 m/s^2 . These values were reasonably close to the measurements in the experimental test [14] where the accelerations on top of the raft were as 4.07 m/s^2 . The results indicate that the chosen boundary conditions, interactions between raft and soil, mesh dimensions, analysis process were fairly reasonable and compatible.

Moreover, because of parameter sensitivity about the dimensions, a model with increased dimensions for the soil was also developed to check whether changing the boundaries and soil dimensions affects the previous results. In this procedure, the soil dimensions for the analysis were set as $28 \times 28 \times 16$ m. Initial sizing of the model was based on the geometry but further analyses were done to achieve the optimum dimension with insignificant change in the results and the minimum required run-time

The obtained results (as illustrated in Table 2) were very close to the results presented above and therefore, the new mentioned dimensions were set as a default for other models and analysis.

Table 2. Comparison of results of Experimental Centrifugal test [14] with the current study.

Experimental centrifugal test [14]	Current Numerical Study	
	Centrifugal test Size (22.5×13.5×12)	Model with larger boundaries (28×28×16)
Maximum horizontal displacement (cm) not reported	6.18	6.21
Maximum horizontal acceleration (m/s ²) 4.07	3.3	3.6

3.2. Pile group foundation

This model was verified with the a case study [12], a pile group with a square cap of 4m×4m and thickness of 1m was put on four square concrete piles with dimensions of 0.5m×0.5m and length of 9m as shown in Figure 3. The pile group system has a 0.5m gap between the soil and raft, therefore the raft and soil are not in contact with one another. The model dimensions were chosen as 28m×28m×16m. Soil properties were the same as above mentioned in the simple raft analysis. The input dynamic loading was similar to the previous section but only the acceleration amplitude was set as 100gal equal to 1m/s² in the prototype scale with the frequency of 1Hz. The calculated results of the acceleration on top of the raft were equal to 4.98m/s² where as the acceleration on the soil surface was measured as 4.2 m/s² and the settlement of the pile group system was about 5cm.

Because of using explicit analysis in dynamic loading all previous stages of analysis should be solved by explicit method as well. Therefore, in order to apply the gravity load of the system, it should be imposed as an equivalent dynamic load.

In other words, the gravity of the system was linearly increased. Meanwhile, horizontal effective stresses ($K_0 \cdot \gamma \cdot h$) were imposed similarly to the system. The result of this type of loading creates an error of almost 3% comparing to the closed form solution which is practically negligible. In addition, because the assumptions of the analysis is based on the structure being on service load, another step of loading is also imposed which is applied as a displacement of 2.5cm or 1 inch which is equal to allowable settlement of the system under its bearing capacity. Results of vertical stress distribution along pile length and inside of the raft after applying the service load stage can be observed in Figure 4. According to the measured values most of the load was transferred from the sides of the piles and a small value was transferred through the pile toes.

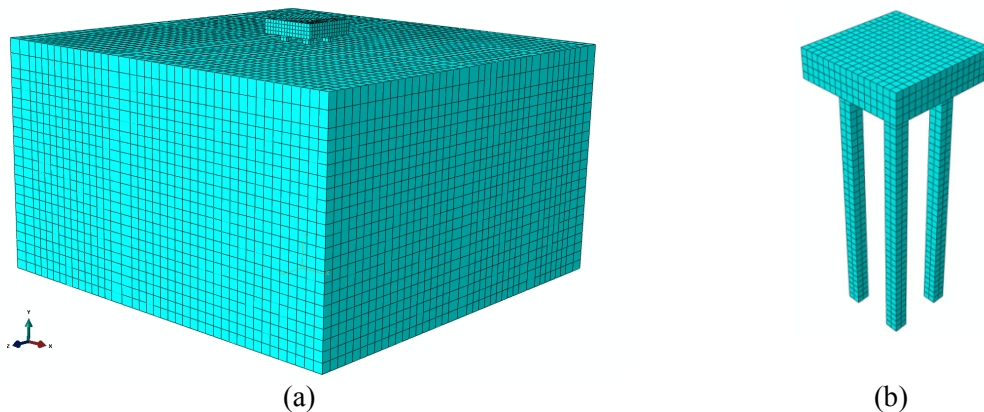


Figure 3. Three dimensional view of pile group model. (a) Pile group and soil; (b) Pile group solely.

Moreover, the horizontal displacement of the system was measured about 10 cm. Therefore, in this analysis very good agreement is observed between the experimental tests from the centrifugal modeling [12] and also the finite element analysis done via ABAQUS. By means, the obtained results in the current study seem very well consistent with the compared paper.

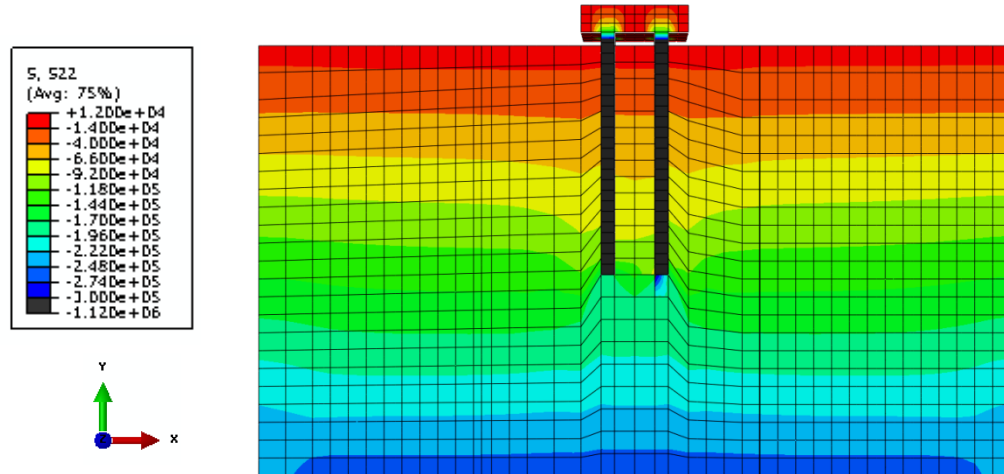


Figure 4. Vertical stress distribution along pile length and inside the raft of the pile group model (N/m²).

3.3. Piled raft foundation

An experimental study [12] was chosen for verification with the same characteristics mentioned in the previous section for the pile group model. Therefore, pile group system with the above mentioned dimensions was put in contact with the surface of the soil in order for the raft to contribute in the load sharing with the piles (as illustrated in Figure 1). In this analysis the sinusoidal acceleration same as the one for the pile group was applied to the piled raft model. The maximum acceleration observed on top of the raft was 3.18 m/s² whereas the acceleration on the soil was about 3.6 m/s². The horizontal displacement of the system was determined in the analysis to be 8.2 cm. Also, the settlement of the system is calculated about 3 cm.

Obtained results from the maximum acceleration response on the top of raft in two models of pile group and piled raft indicate almost 36% decrease in the output accelerations which are shown in Figure 5.

Moreover, as shown in Figure 6 the horizontal displacement in the piled raft system can be even 18% less than a similar pile group model at the same location the piled raft system was almost 18% less comparing to the same pile group model. This indicates suitable performance of piled raft foundations under single frequency dynamic loading comparing to pile group.

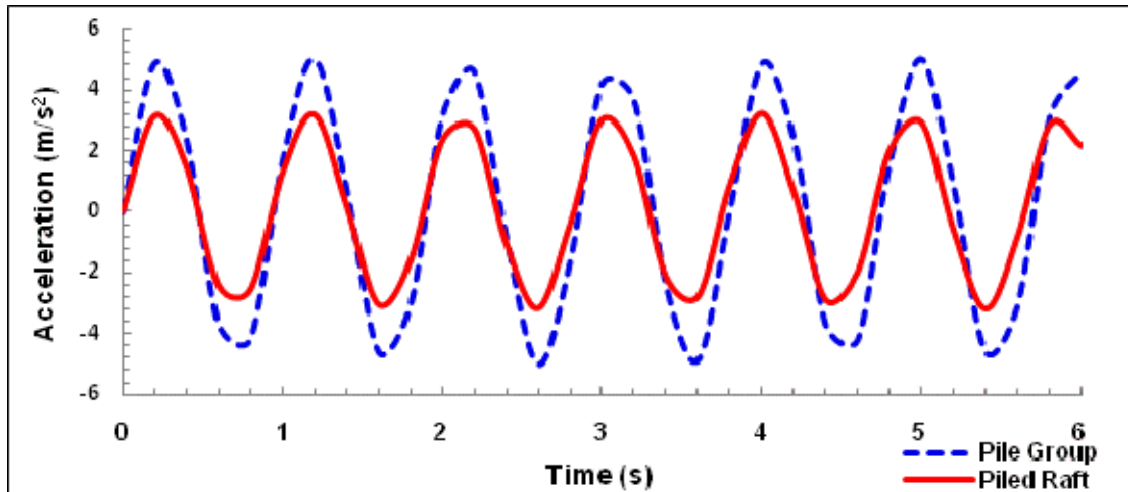


Figure 5. Comparison of acceleration response of pile group and piled raft models under sinusoidal accelerations.

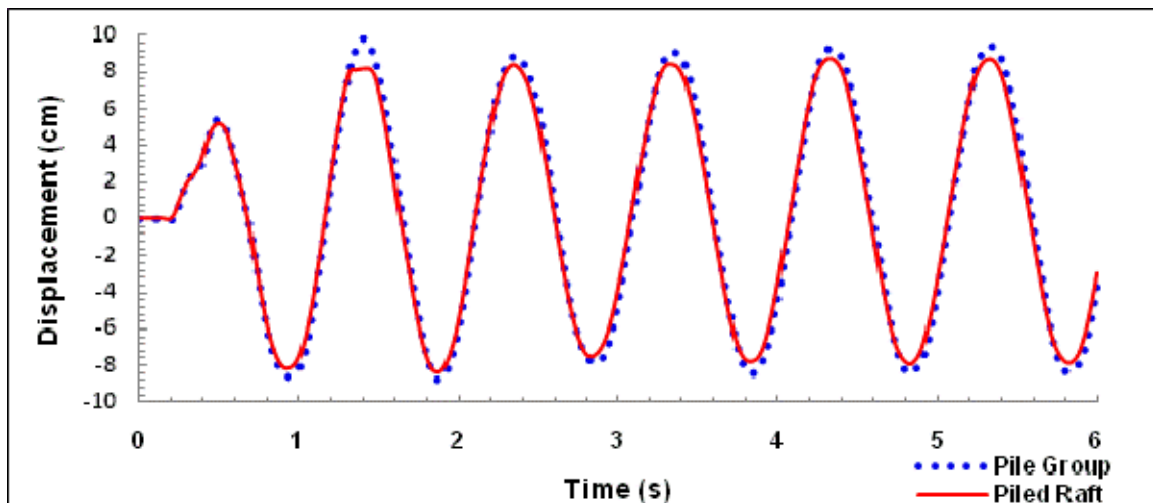


Figure 6. Comparison of horizontal displacements between piled raft and pile group.

The bending moment along pile length for both the pile group and piled raft model is shown in Figure 7. It is important to note that the pile head is fixed to the cap. It can be observed that the peak point is located at the pile head for the pile group system which indicates the maximum bending moment of 336kN.m. Additionally, the maximum bending moment seen for the piled raft model is also located at the pile head and has the value of 154kN.m. This indicates almost 54% reduction of bending moments in the pile by constructing a piled raft foundation instead of a pile group. These values and trends were also measured by [12] and results show that the finite element modeling was consistent to their results.

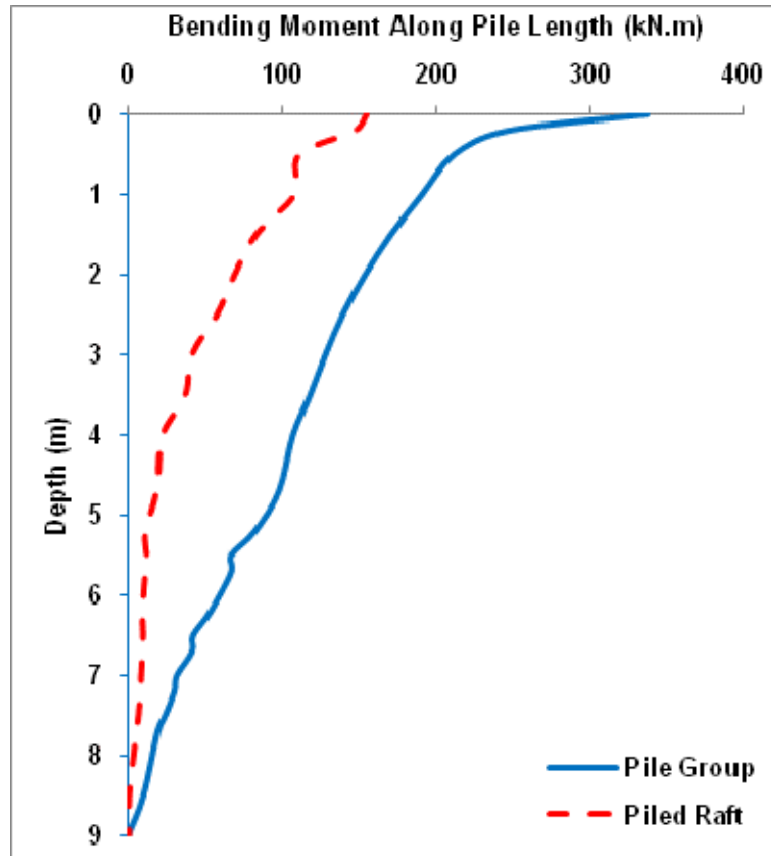


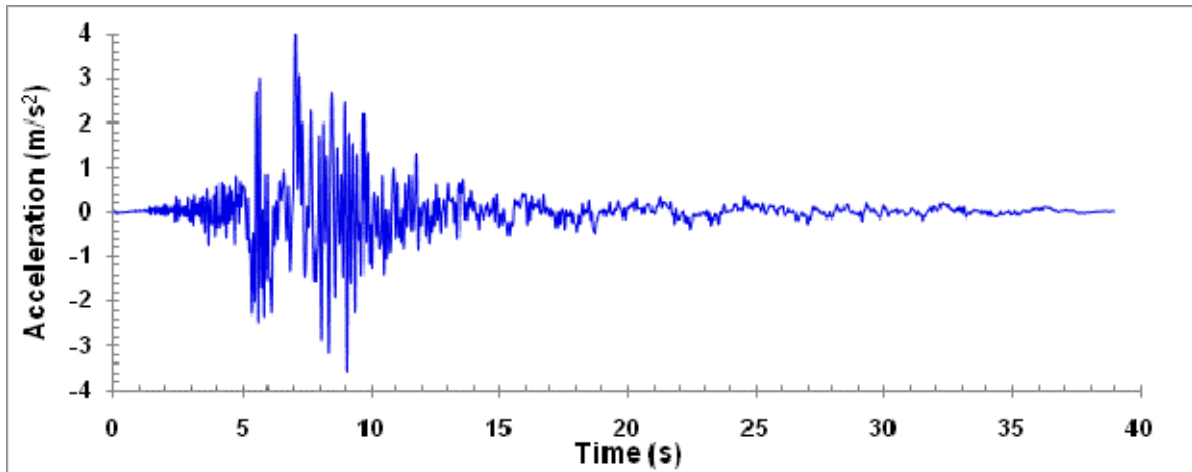
Figure 7. Comparison of bending moments between piled raft and pile groups.

4. Piled raft foundation behavior under earthquake loading

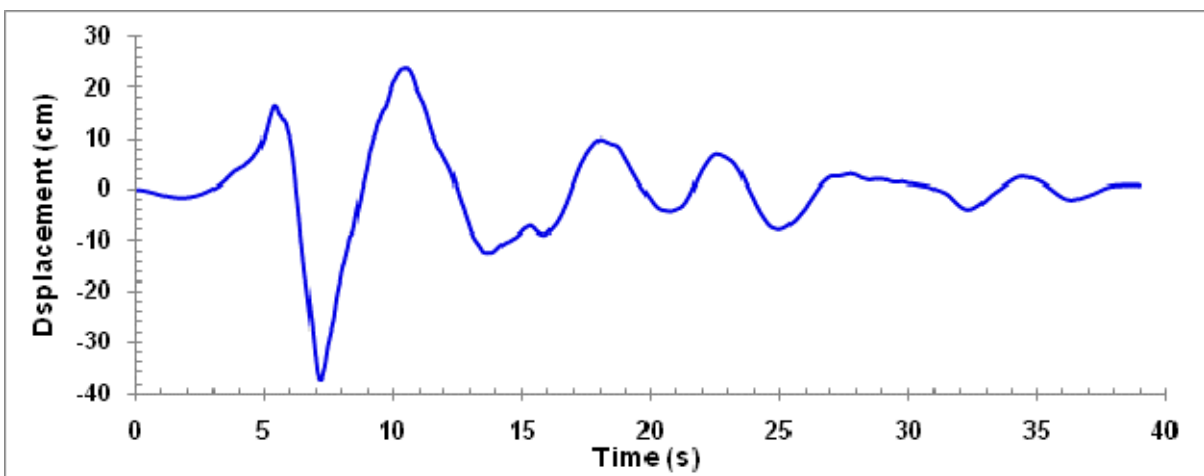
According to the studies based on the seismic behavior of the two foundation systems i.e. pile group and a piled raft which were based on a single frequency sinusoidal loading, an analysis has been done on the same models but under a real earthquake loading with time history. In this case, the El-Centro acceleration time history was chosen for the seismic loading on the piled raft system. According to the previous calibrations, the El-Centro loading can be replaced with the sinusoidal loading regarding the energy content of the earthquake, and the results can be discussed. Since the energy content is unique for an earthquake, the equivalent sinusoidal load can be used for any type of geometry.

In order to apply the El-Centro earthquake time history on the foundation system the input accelerations were corrected under bandwidth filtering methods. Therefore, frequencies less than 0.1Hz and greater than 25Hz were omitted in the filtering. Additionally, utilizing the Seismosignal software the accelerations were transformed into their equivalent displacements. In addition, after the analysis, because of the appearance of some high frequency results, filtering should be applied once more. In this case, the filtering is practical by the solved time steps. According to the ABAQUS documentation this amount should be equal to or greater than $\frac{1}{2\Delta t}$, where Δt is the least time step solved.

The maximum acceleration of the El-Centro earthquake is equal to 0.43g or 4.21m/s² and the maximum horizontal displacement is equal to 37.4cm. The acceleration and displacement time histories of this earthquake are shown in Figure 8.



(a)



(b)

Figure 8. Input earthquake loading of El-Centro earthquake. (a) input acceleration; (b) input displacement.

The obtained results of acceleration of the two piled raft and pile group models are shown in Figures 9 and 10 respectively. The maximum acceleration in the piled raft system was equal to 9.44m/s^2 on the top of raft, also, at the same location for the pile group was equal to 14.2m/s^2 . This indicates about 34% reduction of maximum acceleration of the piled raft model.

Furthermore, according to Figures 11 and 12, the maximum horizontal displacements in the piled raft model and pile group are 40.51cm and 44.25cm respectively. This indicates almost 9% reduction in the maximum horizontal displacement of the piled raft systems. Most importantly, as mentioned above, the maximum acceleration is reduced much more than the value of the horizontal displacement comparing the two models. By means, the combined interaction between pile and raft with soil in the piled raft system is better able to dissipate high frequencies of the earthquake.

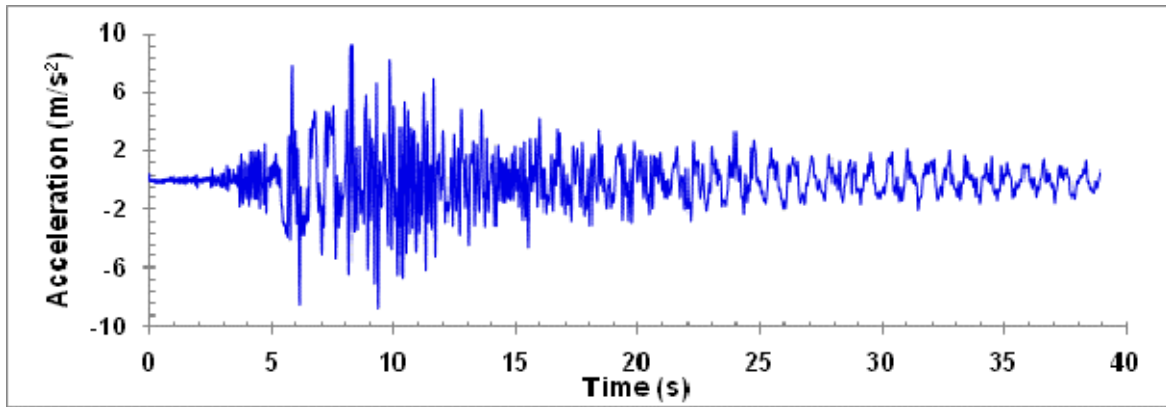


Figure 9. Acceleration response of the piled raft system under El-Centro earthquake loading.

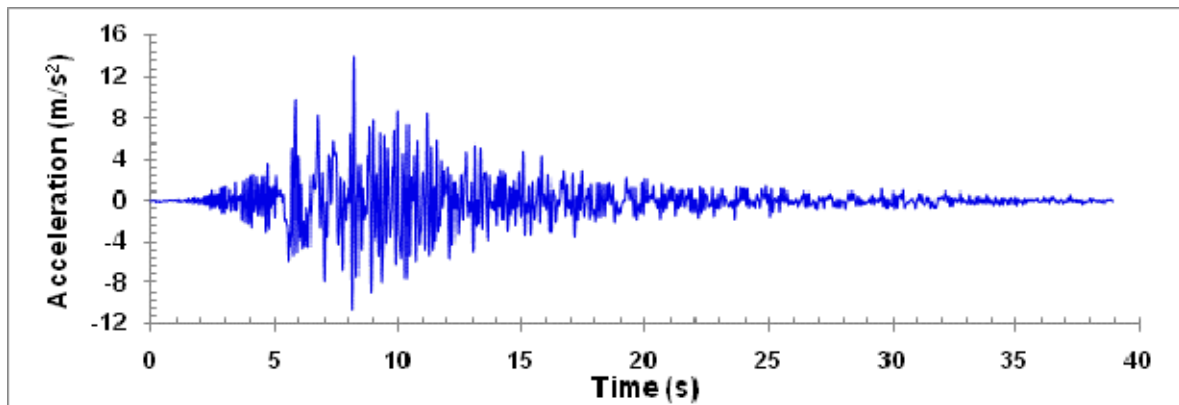


Figure 10. Acceleration response of the pile group system under El-Centro earthquake loading.

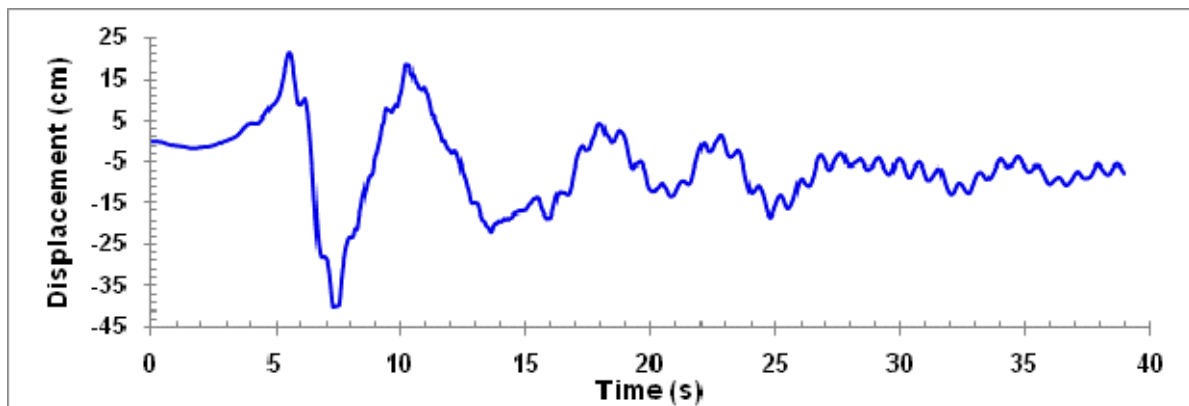


Figure 11. Horizontal displacement of the piled raft system under El-Centro earthquake loading.

In addition, the value of settlement in the piled raft and pile group system are 4.5cm and 10.3cm respectively. These values show the reduction of settlement to a value of 55% in the piled raft foundation comparing to the pile group under the same vertical and earthquake loading. The main reason for this significant reduction in settlement is because of the participation of the raft in load sharing. This phenomenon causes more uniform distribution

of stress in soil deposits. Therefore, the soil experiences much less deformations in the piled raft systems rather than pile groups.

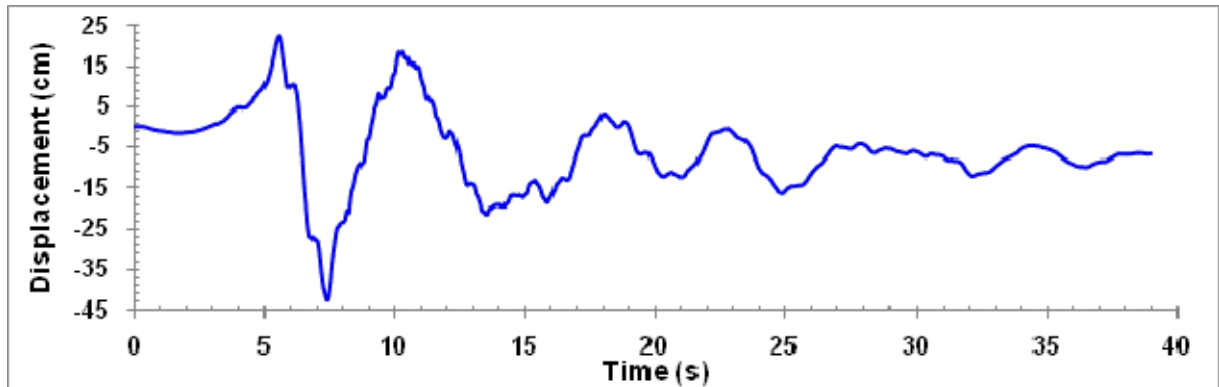


Figure 12. Horizontal displacement of the pile group system under El-Centro earthquake loading.

The maximum bending moment graph of the two models are presented in Figure 13. The bending moment has reached its maximum value in two different time stages for the two foundation systems. The pile group has reached faster to its maximum level of bending moment comparing to the piled raft. In addition, the peak value of bending moment in the piled raft and pile group are 338 and 399KN.m respectively. These values indicate about 15% of reduced bending moments in the piled raft system

Reduction of maximum acceleration, horizontal displacement, bending moment and the filtering of high frequencies in the piled raft foundation system are the most important advantages of the interaction between pile, raft and soil.

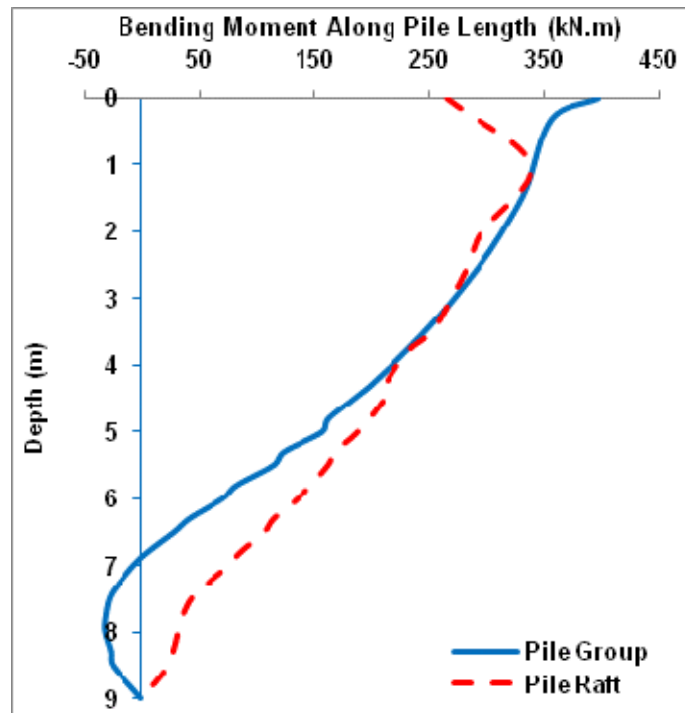


Figure 13. Comparison of bending moments under El-Centro earthquake loading.

5. Summary and conclusions

Traditionally in the calculations of bearing capacity of foundations, only the piles were considered and no emphasis was made on the raft as a load sharing element. To date, according to the advanced numerical analysis, the designer can consider the interaction between a raft (pile cap) and soil in foundation design.

According to an equal volume of excavation needed for the foundation construction, an equal amount of materials necessary and almost an equal cost of construction, the piled raft foundation system have advantages that should be considered in foundation design. With this type of foundation design becoming a common fact, the need of study on its seismic behavior seems necessary. Results in experimental tests of small scale centrifugal modeling imply that the acceleration response of the piled raft foundations rather better than the pile group systems. The objective of this study is to generalize this behavior under earthquake loading and by three dimensional finite element modeling in custom engineering scales. Firstly, a piled raft and pile group models were analyzed and calibrated with an experimental modeling study under single frequency sinusoidal dynamic load. Moreover, by using the time history of the El-Centro earthquake, the seismic behaviors of the foundation systems were observed by finite element modeling.

Results indicate that the seismic behavior of the piled raft foundation system is rather better than the pile group system according to the acceleration response of the piled raft which was 36% less than the pile group. The horizontal displacement of the piled raft was 9% less than the pile group and the maximum bending moment was reduced about 15% for the piled raft system

According to the kinematic studies of these two foundation models, one can expect a rather better response and seismic behavior by the piled raft foundation under earthquake loading. Care should be given to the fact that in saturated soils, in particular fine sands and silts, in which built-up of the excess pore water pressure is very significant under seismic loading, the results may be completely different. Results of this work are restricted to the case of dry soils and saturated soils would require further analyses.

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