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New method for estimation of the scale of fluctuation of geotechnical properties in natural deposits

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Abstract

One of the main distinctions between geomaterials and other engineering materials is the spatial variation of their properties in different directions. This characteristic of geomaterials -so called heterogeneity- is studied herewith. Several spatial distributions are introduced to describe probabilistic variation of geotechnical properties of soils. Among all, the absolute normal distribution was adopted as appropriate distribution which best represents these properties in the horizontal direction.

Variation of geotechnical parameters in the vertical direction is however conceived to follow a deterministic trend. With the aid of random field theory and local average subdivisions (LAS) formulation and using MATLAB Mathworks, virtual data with different correlations were produced and by employing the autocorrelation function, a trend for this function was invoked for different predetermined values of scales of fluctuation. It was found that the autocorrelation function has a deterministic trend as long as the scale of fluctuation is not exceeded. It is concluded that for distances further than the specified scale of fluctuation the behavior is chaotic and this can be an index to calculate the scale of fluctuation of the experimental data.

Keywords: Random field; Autocorrelation; Probabilistic variation; Heterogeneity; Scale of fluctuation.

1. Introduction

Geotechnical engineers are always interested to investigate the real behavior of geomaterials. Nowadays there are several methods available for evaluating properties of such materials. Variables as input parameters in most of these methods are often implemented by assuming that the soil mass is homogenous and the variation of these parameters at different points (which is termed as heterogeneity) is rarely considered because of difficulty and complication. Heterogeneity is a feature of soils in which their parameters are different from one point to another. Almost all natural soils are highly variable in their properties and are

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rarely homogeneous. Soil heterogeneity can be classified into two main categories. The first is lithological heterogeneity, which can be manifested in the form of inclusion of pockets of different lithology within a more uniform soil mass. The second source of heterogeneity can be attributed to inherent spatial soil variability, which is the variation of soil properties from one point to another in space due to different deposition conditions and different loading histories. This heterogeneity in soil parameters can be attributed to the following factors [1]:
(1) Soil inherent spatial variability due to variation in deposition conditions and stress history from one point to another in space;
(2) Deterministic trends in soil properties, such as the increase in soil stiffness with depth due to the increase in confining pressure.

Inherent heterogeneity of first type as mentioned above is mostly related to the variation of properties in the horizontal direction where as the second source mainly emerges to be a trend of soil properties varying deterministically with depth. The aim of this study is to consider variation of geotechnical parameters in horizontal direction which is discussed shortly.

2. Random modeling of soil properties

VanMarcke [2] considering the random behavior of soils in geotechnical issues, concentrated on three parameters: mean, standard deviation (SD) and scale of fluctuation (SF). It is commonplace to use the coefficient of variation (CV) which is the ratio of SD to the mean, instead of SD itself.

2.1. Mean value

As mentioned earlier, variation of properties in horizontal direction constitutes the first source of inherent heterogeneity. As an example of this type of heterogeneity which is due to deposition process, the study of Jaksa et al. [3] can be pointed out in which the scale of fluctuation of cone tip resistance (q_c) in CPT test was evaluated. In this study 51 data for CPT test in horizontal direction and different depths of 3.5, 3.75, 4 and 4.25m were measured. Figure 1 shows the variation of measured data in horizontal direction for different depths.

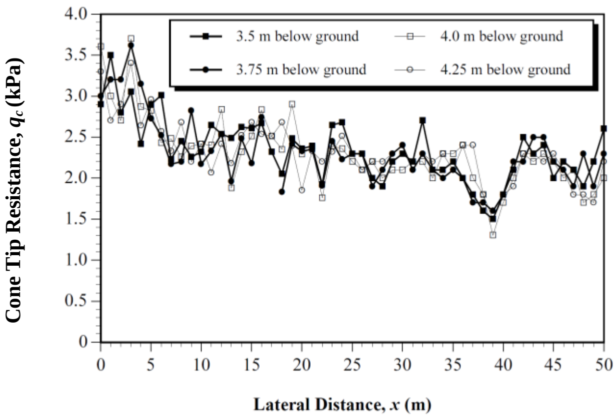


Figure 1. Variation of q_c in horizontal direction for five different depths [3].

Further to the inherent heterogeneity of first type which can be found in both horizontal and vertical directions, variation of soil properties in vertical direction has a deterministic trend as well. For example studies by Jamshidi & Oloomi [4] and Jamshidi & Karimian [5] show that the mean values for stiffness and strength parameters of natural deposits inherit bilinear trends with depth as depicted in Figure 2. The first zone in Figure 2 is related to the upper desiccated zone in which the parameter decreases with depth. The reason behind this extraordinary behavior is embedded within the dependence of the stiffness and strength of fine-grained soils on the moisture content, overconsolidation and effective stress level which has been discussed in detail elsewhere [5]. In the second zone, the parameter increases with depth. The increase in this zone may be linear or parabolic depending on the parameter selected. It can be shown that for undrained shear strength the variation is linear while the stiffness modulus bears a hyperbolic trend. Detailed explanation is found elsewhere [5].

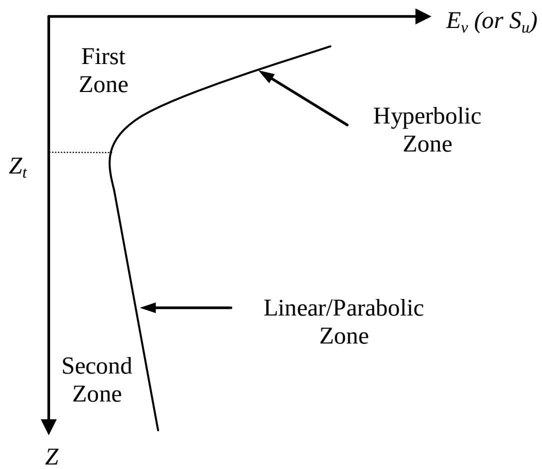


Figure 2. Schematic shape for variation of vertical stiffness modulus [4].

The intersection between the above mentioned two curves which are opposing in variation trend is called transformation depth (Z_t). The transformation depth is the depth at which the decreasing trend of parameter turns to increasing.

2.2. Coefficient of variation

The most commonly used parameter for quantifying the magnitude of uncertain fluctuations in soil properties around their mean values is the coefficient of variation (CV). Lacasse & Nadim [6] reviewing test results from Norwegian Geotechnical Institute (NGI) introduce the CV of soil properties. Phoon and Kulhawy [1] have also studied comprehensively on variation of soil properties arising from inherent variability of these properties.

2.3. Auto-correlation function and scale of fluctuation

The correlation structure of a soil property is described by its correlation function (or equivalently by its spectral density function). In geomechanics, it is often desirable to express the degree of correlation of a soil property by a single parameter. The most commonly used parameter for this purpose is the correlation distance which is a parameter for quantifying coherence of soil properties and a distance over which soil parameters exhibit strong correlation and beyond which, they may be treated as independent random variables [7].
The autocorrelation function (ACF) is the variation of the autocorrelation coefficient, ρ_τ, with lag, τ, as expressed by

$$\rho_\tau = \frac{c_\tau}{c_0} \tag{1}$$

Where: c_τ is the autocovariance at lag $\tau = Cov(X_i, X_{i+\tau}) = E[(X_i - \overline{X})(X_{i+\tau} - \overline{X})]$, X_i is the value of property X at location i; $\overline{X}$ is the mean of the property X; $E[...]$ is the expected value; c_0 is the autocovariance at lag 0; $c_\tau = c_{-\tau}$ and $\rho_\tau = \rho_{-\tau}$.
It is not possible, however to know *neither* c_τ *nor* ρ_τ with any certainty, but only to estimate them from samples obtained from a population. As a result, the sample autocovariance at lag

τ, c^*_τ, and sample autocorrelation at lag τ, r_τ, are generally evaluated. The sample autocorrelation function is the graph of r_k for lags τ = 0, 1, 2, ... T, where T is the maximum number of lags (data intervals) allowable (generally, K = N / 4 [8], where N is the total number of data points). The sample ACF at lag τ, r_τ , is generally evaluated using:

$$r_\tau = \frac{\sum_{i=1}^{N-k}(X_i - \overline{X})(X_{i+\tau} - \overline{X})}{\sum (X_i - \overline{X})^2} \tag{2}$$

Examples of auto-correlation functions commonly used in geotechnical engineering for soil properties are presented by VanMarcke [9] and Li and white [10].

Table 1. Different ACFs used in Random Field theory [9, 10].

Model No.	Autocorrelation Function
1	$\rho_\tau = \begin{cases} 1 - \frac{\lvert\tau\rvert}{\theta} & for\ \lvert\tau\rvert < \theta \\ 0 & for\ \lvert\tau\rvert > \theta \end{cases}$
2	$\rho_\tau = e^{-2\lvert\tau\rvert/\theta}$
3	$\rho_\tau = e^{-\pi(\lvert\tau\rvert/\theta)^2}$
4	$\rho_\tau = e^{-4\lvert\tau\rvert/\theta}\left(1 + \frac{4\lvert\tau\rvert}{\theta}\right)$

Figure 3 shows a graphical representation of different autocorrelation models presented in Table 1. The second model which is known as Markovian correlation function was used throughout this study.

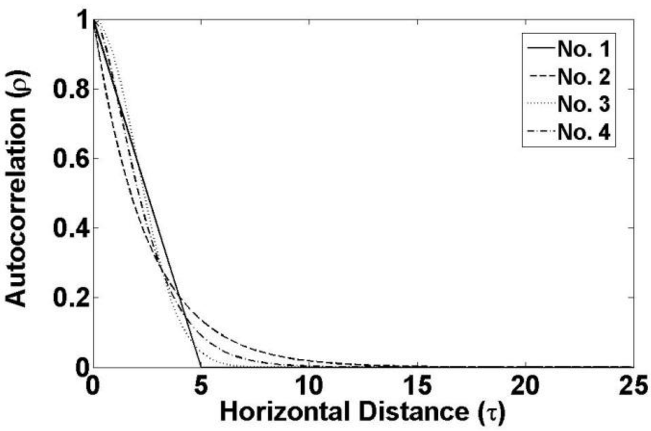


Figure 3. Different autocorrelation functions (ACF).

Jaksa et al. [11] showed that the scale of fluctuation, θ_i, of some parameter *i*, can be estimated relatively simply by evaluating Bartlett's distance, which is the correlation distance defined by times series analysis. It is calculated by determining the lag at which the sample ACF first intersects Bartlett's approximation or limits, as given below:

$$r_\tau = \pm \frac{1.96}{|N|} \tag{3}$$

For instance, variation of ACF with horizontal distance for data of q_c presented earlier in Figure 1 is drawn in Figure 4. These curves intersect Barttlet's limit in distances between 0.9 and 1 which is assumed as the scale of fluctuation of this data.

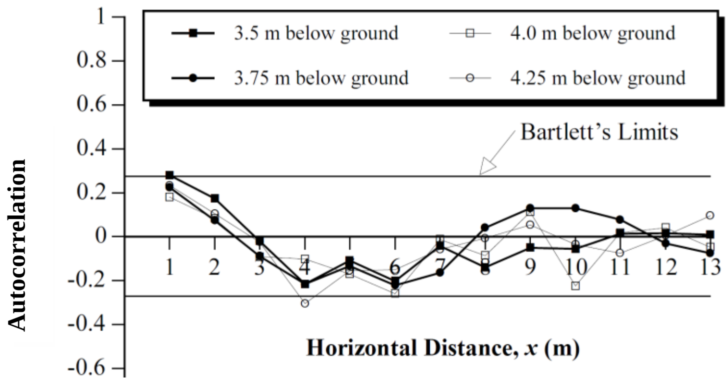
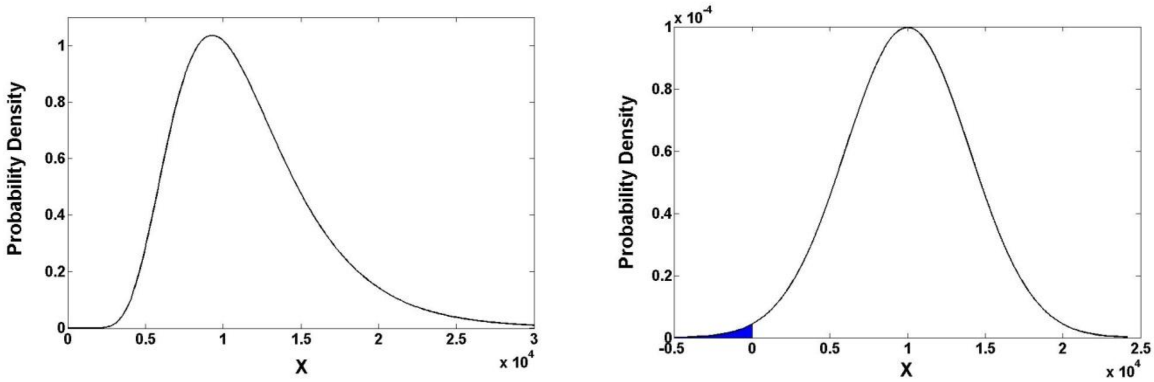


Figure 4. Variation of ACFs for q_c with horizontal distance [2].

2.4. Probability distribution

Most soil properties exhibit skewed, non-Gaussian distributions and each soil property can follow different probability distributions for various materials and sites [12]. Due to physical reasons, they always have to follow distributions defined for strictly non-negative values of the soil properties. While there is no clear evidence pointing to any specific non-Gaussian PDF for each soil parameter, one condition that has to be satisfied for the assumed PDF is to have a lower bound. This restriction leads to the use of truncation distribution which arises in practical statistics in cases where the occurrences are limited to values which lie above or below a given threshold or within a specified range.

Most soil properties like deformation modulus, undrained shear modulus, etc inherit zero cutoff values. Lognormal distribution meets the above mentioned requirements and is commonly used for this purpose. This distribution is asymmetric (positively skewed) and it's more probable for random variable to take values greater than its average. Figure 5 (a) shows the probability density function of lognormal distribution for parameter X with mean, 10^4 and CV, 40%. The probability for this parameter to become greater than its mean value is 57%.



a) b)

Figure 5. Probability density function for parameter X with μ=10^4 kPa and CV=40% (a) lognormal distribution (b) normal distribution.

The other limitation of lognormal distribution used by prior researchers is that in this distribution correlation between logarithmic data doesn't guarantee the real correlation between data itself. In other words if two near points bear small difference in logarithmic scale, it means that they may have a large difference in true scale which may become unacceptable. This limitation may be overcome by taking more real and appropriate distribution for soil parameters.

If we take normal (Gaussian) distribution and assume the mean and CV similar to the above example, the probability for this parameter to be negative is 0.62% which is very small and trivial (Figure 5(b)). This means that by applying an absolute operator to the generated data we can preclude appearance of negative values without any major change to the distribution shape. The discussion behind the issue of distribution type and probability distribution function (PDF) is beyond the scope of this paper. Some researchers have recently studied the effect of different statistical distribution of geotechnical parameters on the behavior of geotechnical structures. Results indicate the type of distribution considered for characterization of the random nature of geotechnical parameters can have a significant impact on computed results [13].

3. Data generation

The data generation in this study was performed by assuming an arbitrary parameter, X with one-dimensional distribution. Random field theory of VanMarke [2] and local average subdivisions (LAS) technique [14] was utilized in order to produce data with specified statistical parameter which is simultaneously spatially correlated. An absolute normal distribution was employed to make a 1D realization of 100 data with zero mean and unit standard deviation (SD). Any arbitrary data set can then be generated by multiplying the above mentioned standard data set to the desired standard deviation and adding with the required average. For example if we aim at generating a data set with average 10^4 and CV of 40% (SD = 4×10^3), it is first needed to produce a standard data set, X as explained. Then the target data set, X' will be obtained as follows:

$X' = 0.4 \times 10^4 X + 10^4 = (0.4 X + 1) \times 10^4$

It is expected from statistics that the summation and multiplying operations will not change the correlation structure.

Different values of 1, 2, 4, 6, 8, 16 and 20 for the scale of fluctuation were selected and LAS technique was employed so as to generate realizations for each case with prescribed distribution type, mean and standard deviation values.

LAS theory follows a recursive fashion where in stage zero a global average with absolute normal distribution, zero mean and unit standard deviation is created. In stage one, the field is subdivided into two equal parts whose average equals to the parent value. In stage two, two absolute normally distributed values are generated whose means and variances are selected so as to satisfy three criteria: a) that they show the correct variance according to local averaging theory; b) That they are properly correlated with one another; c) That they average to the parent value and so on in this fashion. More details on the LAS realization technique is available elsewhere [14, 15].

A unit sampling interval was assumed. However it is believed that the scale of fluctuation is strongly depended on the distance over which it is estimated and sampling interval in other words.

Fenton [16] observed that soil properties seem to be fractal in their nature, hence a fluctuation scale will become smaller/ larger as the domain decreases/ increases. For example sampling soil properties every 5cm over 2m will likely yield an estimated scale of fluctuation of about 20cm, while sampling every 1km over 1000km will likely yield an estimate of 200km.

The process of data generation is such that 100 realizations of the studied parameter are produced, autocorrelation function (ACF) for each of the realizations is calculated and then an average ACF is reported in order to discuss on correlation structure. ACFs are calculated and drawn against the lag distance. The scale of fluctuation is then calculated to compare with the assumed correlation distance based of which the data was generated and that of calculated by Bartlett's method (Figures 6, 8 and 9). In these figures θ_{exp} is the scale of fluctuation based of which the data was generated, θ_{cal} is the calculated scale of fluctuation based on current method and θ_{Bat} *is* that of calculated by Bartlett's method. Looking into the ACF plots for different assumed scale of fluctuations it is evident that the ACF follows a deterministic trend when there is still enough correlation between the data nearby. Figure 7 indicates the schematic variation form of the ACFs.

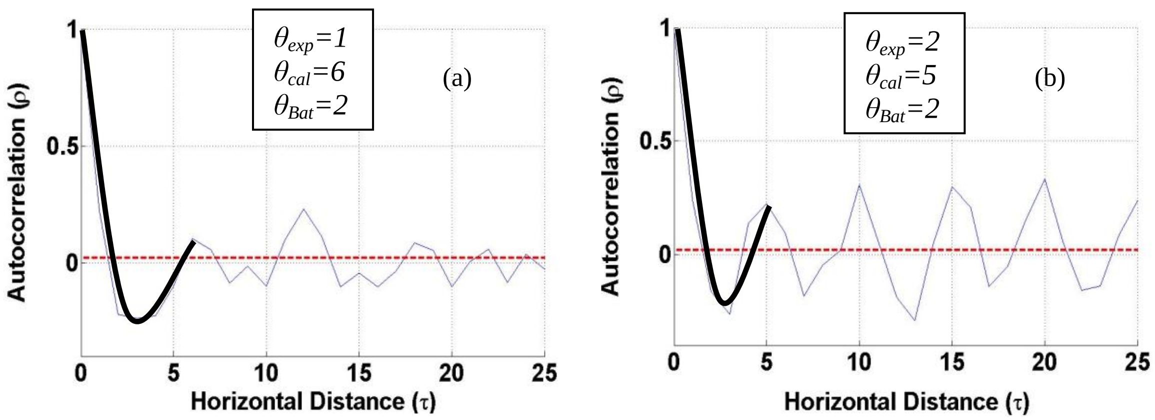


Figure 6. Variation of ACF with lag distances for a) θ=1 b) θ=2.

It is seen that when the lag distance is less than the scale of fluctuation, a deterministic trend exists which is in the form of weak-form parabola and it becomes wider by increasing of the scale of fluctuation or correlation distance. For lag distances beyond the correlation distance, the ACF is random in nature and no meaningful trend is found.

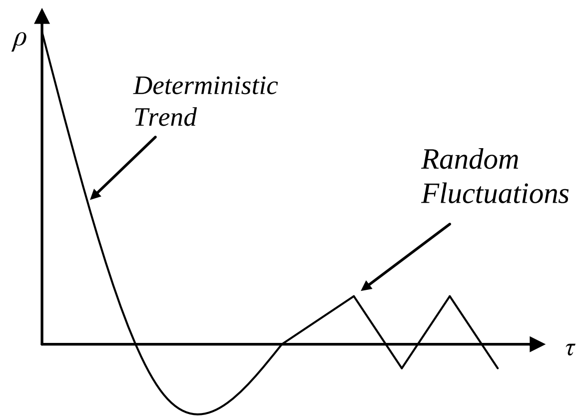


Figure 7. Schematic shape for the variation of ACF with lag distance.

Comparison of the plots provided in Figures 6, 8 and 9, it is obvious that for low scale of fluctuation (θ_{exp} = 2), the calculated correlation distance (θ_{cal}) deviates from the expected value (θ_{exp}) and this deviation vanishes for higher scales of fluctuation (θ_{exp} > 2).

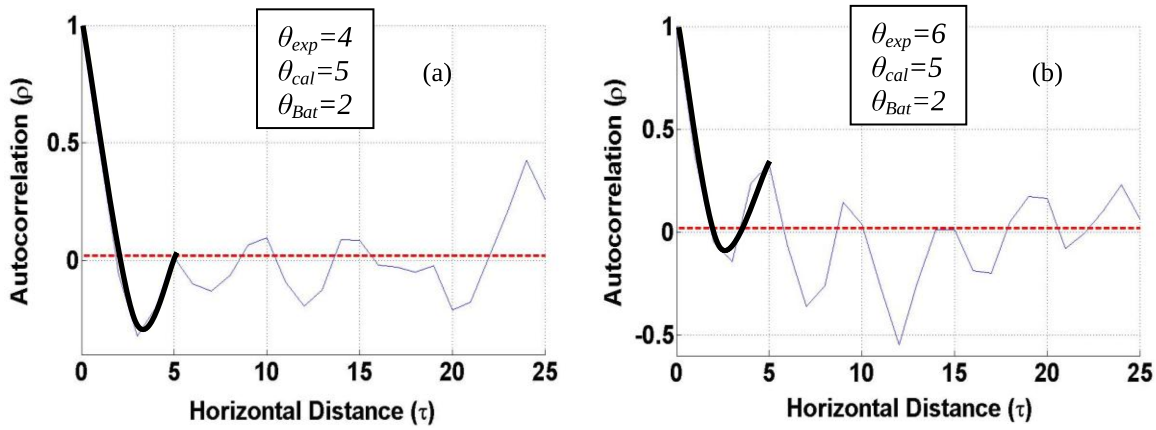


Figure 8. Variation of ACF with lag distances for a) θ=4 b) θ=6.

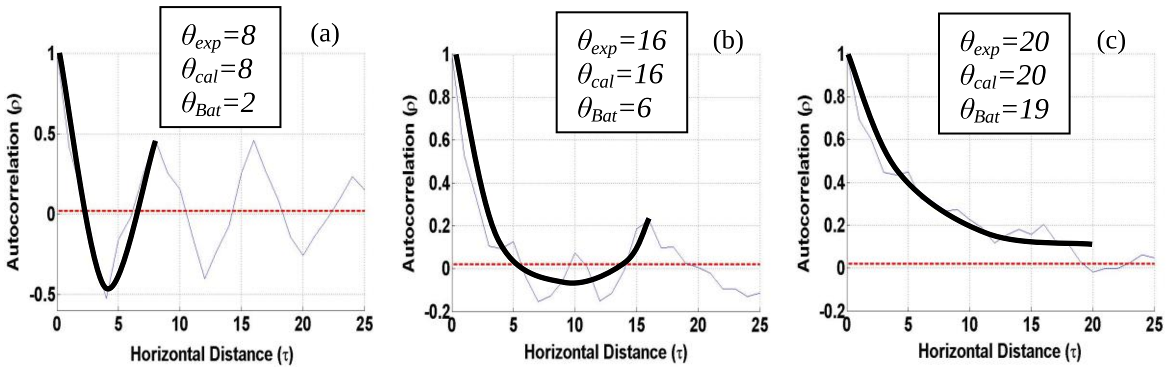


Figure 9. Variation of ACF with lag distances for a) θ=8 b) θ=16 c) θ=20.

A reverse order of conformity is found for Bartlett's limit. This means that Bartlett's approximation gives only acceptable prediction when the scale of fluctuation is small.

However although the introduced method proved efficient in providing information about the scale of fluctuation, it should be noted that the proposed model is a conceptual method and a clear distinction between deterministic and random components of the ACF profile is somehow qualitative and needs experience. The general form of the weak parabola is to be:

$$\rho = \left(\frac{a}{\theta^2}\right)\tau^2 - \left(\frac{b}{\theta}\right)\tau + 1 \tag{4}$$

where *a* varies between 2 and 4.5 and *b* between 1 and 2.5.

For better indication the application of the proposed method of computing the scale of fluctuation (SF) of soil properties, it is applied to some real cases. Figure 10 shows the superimposed deterministic trend to the data shown earlier in Figure 4. Earlier it was seen that SF value estimated by Bartlett's limit was found to be 1m. By fitting a deterministic trend to the ACF as proposed in this study, SF value is estimated to be 9m. Figure 11 indicates ACF of measure $q_c^{0.74}$ data. The scale of fluctuation (θ) for these data is 5.5m which is in conformity to what stated by Fenton [16].

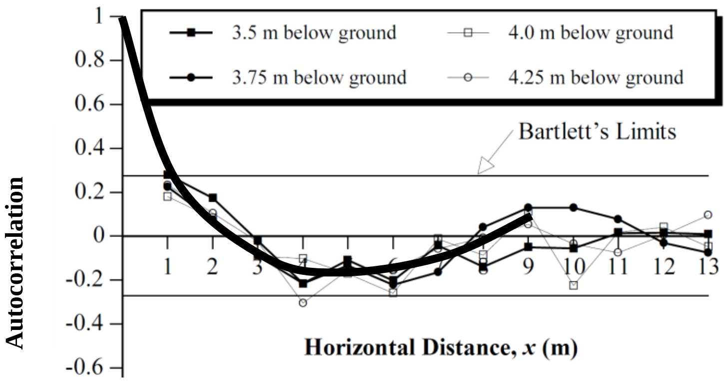


Figure 10. Variation of ACFs for q_c and fitted deterministic trend with horizontal distance (Modified from Jaksa et al. [3]).

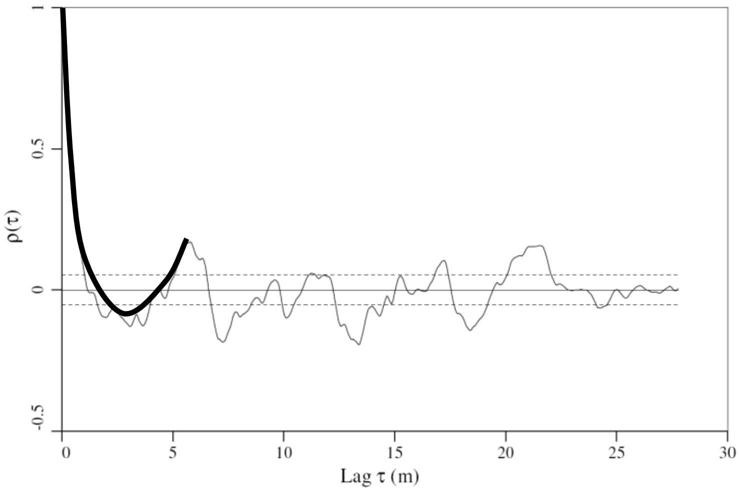


Figure 11. ACF for a $q_c^{0.74}$ record of length 27.8 m and fitted deterministic trend (Modified from Fenton [16]).

4. Conclusion

Heterogeneity as a phenomenon inherent to geomaterials is considered in this study. Since variation of the behavior of geomaterials in vertical direction is influenced by the stress level and overconsolidation ratio, a deterministic trend is expected to dominate the profile of most important geotechnical parameters in vertical direction. For this reason, stochastic variability modeling was limited to one-dimension (horizontal direction). For this purpose random field theory, autocorrelation function and required parameters were introduced; an absolute-normal distribution was selected for data generation. With the aid of local average subdivisions theory, data with predetermined scale of fluctuation values were generated and the average autocorrelation functions, ACFs for 100 realizations were plotted. It was shown that autocorrelation function has a deterministic trend when the lag distance is less than the scale of fluctuation and the end point of this trend indicates the scale of fluctuation value and this deterministic trend is indeed a weak-form parabola and it becomes wider by increasing the scale of fluctuation. For distances beyond the scale of fluctuation value, this function has random variation with no meaningful trend.

Scale of fluctuation value which is calculated by superimposing the deterministic trend to the autocorrelation function, proved to be well predicted by the model proposed in this study as long as the expected correlation distance is not too small (> 2m).

However, although the proposed model is conceptual and provides qualitative information about the scale of fluctuation, it has addressed some important features of the autocorrelation function, the most important of which is that it has two distinct portion referred to as deterministic and random variation respectively. It should be noted that the model introduced herewith is not a curve fitting process and only a weak-form parabola was superimposed to the drawn autocorrelation function.

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