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Uncertainty in fundamental natural frequency estimation for alluvial deposits

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Abstract

Seismic waves are filtered as they pass through soil layers, from bedrock to surface. Frequencies and amplitudes of the response wave are affected due to this filtration effect and this will result in different ground motion characteristics. Therefore, it is important to consider the impact of the soil properties on the evaluation of earthquake ground motions for the design of structures. Soil properties in a heterogeneous soil layer are affected by a series of uncertainties. Inherent variability of shear modulus contributing most to the overall uncertainty is described and modeled in this paper using deterministic and stochastic field representations. This paper highlights the importance of the effect of the stochastic components of the heterogeneity of shear modulus on response behavior of natural alluvial deposits in comparison to the deterministic consideration. The results emphasize that stochastic heterogeneity has significant effects in dynamic properties of natural alluvial deposits and its negligence would result in an over estimation of the fundamental frequency of spatially variable alluvial deposits.

Keywords: Natural frequency; Inherent variability; Shear modulus; Stochastic fields; Natural alluvial deposits.

1. Introduction

Geotechnical earthquake engineering deals with many kinds of uncertainties which should be considered in the computation of site earthquake response. These uncertainties are related mainly to earthquake input which is caused by earthquake source mechanism, transmission path, and also to, soil properties which vary from place to place within deposits [1]. The uncertainty in field data is introduced by the inherent soil heterogeneity, namely spatial variability of soil properties within so-called homogeneous soil layers.

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A study of horizontal ground acceleration response spectra was published by Seed et al. [2]. This investigation included 104 accelerograms from sites in the United States and other countries where the soil conditions were known in some detail. Figure 1 shows the response spectra for buildings normalized with respect to the peak ground acceleration. It is indicated that tall buildings founded on deep or soft alluvium may be subject to seismic forces several times larger than similar buildings on rock, if the peak ground acceleration is the same in both cases. Therefore, observed damage patterns and the statistical analysis of recorded motions suggest that both stiffness and depth of soil stratum affecting the natural frequency should be considered in the seismic design of structures, especially in the long-period range.

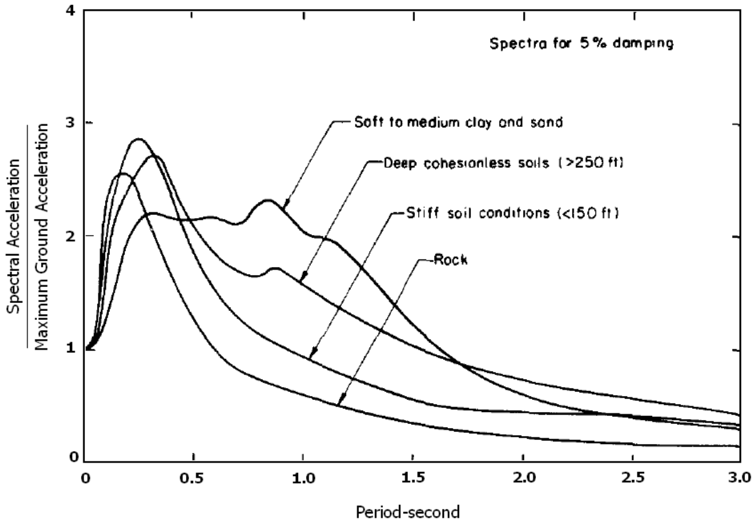


Figure 1. Average acceleration spectra for different site conditions [2].

Simplifications provided by former researchers (Dobry et al. [3] for example) are all to make analytical solution possible to the heterogeneous cases. The attention has recently been more focused on the realistic variation profile of the dynamic parameters of natural alluvial deposits. Towhata [4] pointed out this matter efficiently by incorporating a nonlinear variation pattern for shear modulus. He concluded that more energy can reach the ground surface than in the more simplified conventional way of calculation. Later on and very recently Rovithis et al. [5] explored the seismic response of inhomogeneous soil deposits analytically by means of one-dimensional viscoelastic wave propagation theory. Their work comprised of a continuously inhomogeneous stratum over a homogeneous layer of higher stiffness by employing a generalized parabolic function to describe the variable shear wave propagation velocity in the inhomogeneous layer. The above mentioned studies are based solely on the deterministic variation of soil dynamic properties. However, the geological configurations and the material properties of soil deposits such as density, elastic moduli and damping coefficients are not always known with sufficient accuracy to justify a deterministic analysis. Consequently, uncertainties in the properties of the medium will result in uncertainties in the predicted responses. In other word, deterministic descriptions of the spatial variability of soil properties are not always feasible, and the sufficiently large degree of disorder exhibited, leads to the use of statistical methods in describing their distribution within a "statistically homogeneous" soil zone [6].

Nour et al. [7] dealt with the numerical simulation of heterogeneous soil profile and its behavior under uniform seismic environment. Soil properties of interest were shear modulus, damping ratio and Poisson's ratio, modeled as spatially random fields. In this frame, the seismic response was carried out via Monte Carlo simulations combined with deterministic finite element method. The analysis integrated the influence of, coefficient of variation of the three soil properties, the inter-property correlation coefficients, as well as horizontal and vertical correlation lengths. Results of this analysis indicated that heterogeneity highly influences the behavior of the soil profile. Obtained results indicated that the shear modulus and damping ratio are of prime importance and Poisson's ratio variability can be neglected in a dynamic analysis of a soil profile.

The latest study in the stochastic field belongs to Sadouki et al. [8] by presenting a unified formulation of an analytical method for evaluating the response of a random soil medium to surface or earthquake excitations. Specifically, their study focused on the case of a horizontally stratified layered soil profile. Soil properties, mass density and shear modulus of each layer were modeled as spatial random fields to examine the effects of stochastic variations of mass density and shear modulus on the amplification function.

The random fields for shear modulus are generated using simulations by the Monte Carlo method. This method consists of performing a set of probabilistic realizations of the medium, used hereunder to predict the transfer function and the extreme acceleration at ground surface via deterministic calculation for each realization, and proceeding thereafter to the statistical treatment of the obtained results.

The scope of this paper is to investigate the effect of inherent variability of soil profile on natural frequency contents of alluvial deposits. Soil property of interest is shear modulus modeled herein as spatially random field. Shear modulus was modeled using the lognormal distribution. This choice is motivated by the fact that lognormal distribution enables realization of positive shear moduli with different levels of variability. In this regard, the dominant natural frequency of heterogeneous soil stratum was investigated by developing a 'FISH' code in FLAC and conducting a so called "sweep" test. Monte Carlo simulation approach was used in generation of *2D* log-normally distributed correlated random fields.

Frequency content of a system with multiple degrees of freedom (*MDOM*) consists of different modal frequencies. There all always a few frequencies which are dominant frequencies among all. A couple of first frequencies mainly control the resonance behavior of the system. This paper focuses on the first five modal frequencies of a homogenous system and investigates the fitness and closeness of the dominant frequency of a stochastic heterogeneous system to the modal frequencies of homogenous system. Analyses highlights the effect of shear modulus variability introduced as the coefficient of variation and its correlation distance, on the resonance behavior and natural frequency contents of heterogeneous soil stratum in comparison to homogenous condition.

2. Sources of uncertainty in geotechnical soil parameters

There are three primary sources of uncertainty in geotechnical design parameters: inherent soil variability, measurement error, and transformation uncertainty (Figure 2). Inherent variability is the consequence of natural geologic processes that continually modify the soil mass in situ. Measurement error results from equipment, test-operators, and random test effects during measurements. Transformation uncertainty is introduced when field or laboratory measurements are "transformed" into design soil properties with empirical or other correlation models. Among these three sources of uncertainty, only the inherent soil variability is taken into consideration in this study.

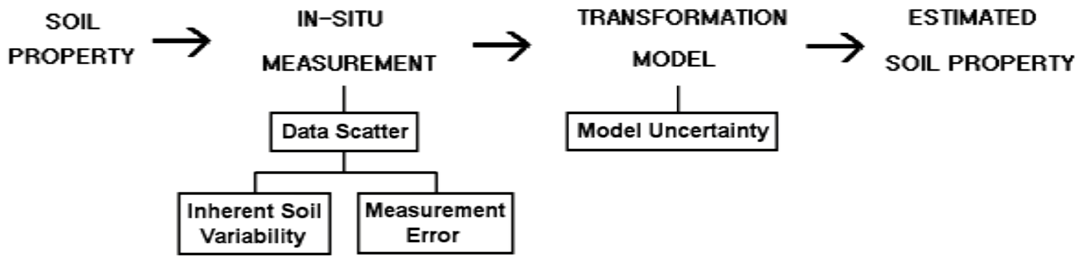


Figure 2. Uncertainty in soil property estimates modified from Kulhawy [9].

Although the inherent variability is common in soil layer which is homogenous in terms of composition, in majority of cases in geotechnical engineering, one will encounter with soil strata with different lithological origins. This type of variability called lithological heterogeneity results from the formation of soil layer from decomposition of different parental material. So, along with the inherent variability in natural alluvial deposits, there is generally another source of variability manifested in the form of soft/stiff layer embedded in a stiffer/softer media or the inclusion of pockets of different lithology within a more uniform soil mass, but this is excluded in this paper.

Inherent variability in geotechnical properties can be modeled by equation (1) in which a depth dependent geotechnical property, ξ is decomposed into the deterministic component, $t$and the fluctuating component, w that totally represent the inherent soil variability. Figure 3 shows schematically the inherent variability.

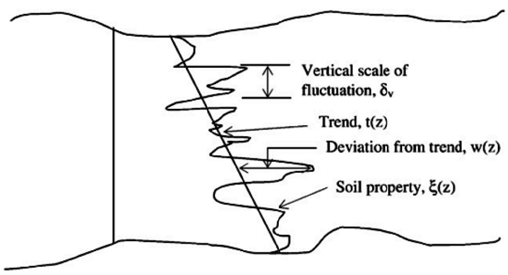


Figure 3. Statistical description of inherent variability [10].

$$\xi(z) = t(z) + w(z) \tag{1}$$

If $w(z)$ is considered to be statistically homogeneous, the mean and variance of *w(z)* are independent of depth, and the correlation of *w(z)* signals at two different depths is a function of their spatial separation rather than their absolute locations [10].

Quantitative assessment of the uncertainty embedded in soil properties requires the use of statistics, as well as probabilistic modeling to process data from laboratory or in situ measurements. Probability theory is useful in modeling the observed behavior of a variable parameter if a set of measurements are available. Any quantitative geotechnical variability relies on sets of measured data which are often limited in size and hence, it is referred to sample statistics. The uncertainty in the measured data is expressed in terms of sample mean (μ) and variance (σ^2) evaluated from the following expressions.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{2}$$

$$\sigma^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{3}$$

where n is the total number of samples.

A useful dimensionless ratio can be calculated by normalizing σ with respect to the "local" mean soil property, μ obtained from the depth varying trend function, *t(z)*. This ratio is called the coefficient of variation, COV.

$$COV = \frac{\sigma}{\mu} \tag{4}$$

An extensive literature review was made by Phoon and Kulhawy [10] to estimate the typical COV values of inherent soil variability. However, this task was complicated because most *COV*s reported in the geotechnical literature are based on total variability analyses, as illustrated in Figure 3. Therefore, the reported *COV*s may be considerably larger than the actual inherent soil variability because of four potential reasons: (1) soil data from different geologic units are mixed, (2) equipment and procedural controls are generally insufficient, (3) deterministic trends in the soil data are not properly removed, and (4) soil data are gathered over a long period of time with different testing techniques.

The lower bound for the COV of inherent variability of undrained shear strength was reported by Phoon and Kulhawy [10] to remain relatively constant at about 10% over the range of mean values studied.The upper bound on the COV seemed to decrease with increasing mean. However, it was found not to exceed 50%. If someone could assume the undrained shear modulus, a linear function of a the undrained shear strength as a single random variable, the First Order Second Moment (*FOSM*) concept would lead to the conclusion that the mean and standard deviation of the undrained shear modulus are the same function of undrained shear strength. It could then be expected that the typical values of COV_G for undrained shear modulus lie in the range of *10–50%*. Current study solely considers a wider range of 10-100% for parametric study purpose. Other parameters like, density and Poisson's ratio are maintained at constant values because they do not show strong variation according to literature review [10].

3. Correlation structure

The property variations of the in-situ soil represented by the mean value, coefficient of variation and correlation distance influence the likely parameters for design. In the present study, soil shear modulus G is considered as random variable and assumed to be a log-normally distributed value represented by parameters mean, μ_G standard deviation,$\sigma_G$and spatial correlation distance, δ_V which represents a length over which significant correlation in a specific soil property is still observed. A large autocorrelation distance value implies that the soil property is highly correlated over a large spatial extent, resulting in a smooth variation within the soil profile [11]. Use of log-normal distribution is appropriate as the soil properties are non-negative and the distribution also has a simple relationship with normal distribution. A log-normally distributed random field is given by:

$$G\ (\tilde{x}\ _i) = exp\{\mu_{ln\,G}(\tilde{x}) + \sigma_{ln\,G}(\tilde{x}).\,N_i\ (\tilde{x})\} \tag{5}$$

where $\tilde{x}$ is the spatial position at which G is desired. $N(\tilde{x})$ is a normally distributed random field with zero mean and unit variance. The values of $\mu_{\ln G}$ and $\sigma_{\ln G}$ are determined using log-normal distribution transformations given by:

$$\sigma_{ln\,G}^2 = ln\left(1 + \frac{\sigma_G^2}{\mu_G^2}\right) = ln(1 + COV_G^2) \tag{6}$$

$$\mu_{ln\,G} = ln\,\mu_G - \frac{1}{2}\sigma_{ln\,G}^2 \qquad (7)$$

The correlation structure is assumed to follow an exponentially decaying correlation function as given by:

$$\rho_G(\tau) = exp\left(-\frac{2\tau}{\delta_V}\right) \qquad (8)$$

where $\tau = |\tilde{x}_1 - \tilde{x}_2|$ is the absolute separation distance between two points and δ_V is the spatial correlation distance. The correlation matrix is decomposed into the product of a lower triangular matrix and its transpose by Cholesky decomposition.

$$L.L^T = \rho_G \qquad (9)$$

Given the matrix L, correlated standard normal random field is obtained as follows [12]:

$$N_i = \sum_{j=1}^{i} L_{ij}Z_j, \qquad i = 1,2,\dots,n \qquad (10)$$

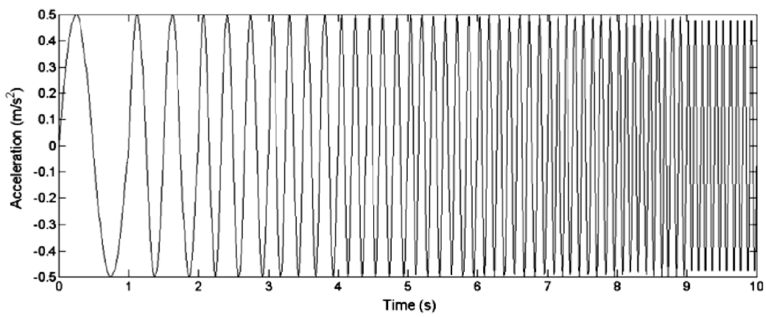
where Z_j is the sequence of independent standard normal random variables and N_i represents the correlated field. In present study, results are presented assuming that the soil has anisotropic correlation structure; therefore the correlation distance is not assumed the same in both horizontal and vertical directions. The assumption of anisotropy leads to the adoption of equation (11) for the correlation matrix.

$$\boldsymbol{\rho_G(\tau) = exp\left(-2\sqrt{\frac{\Delta x^2}{\delta_H^2} + \frac{\Delta z^2}{\delta_V^2}}\right)} \qquad (11)$$

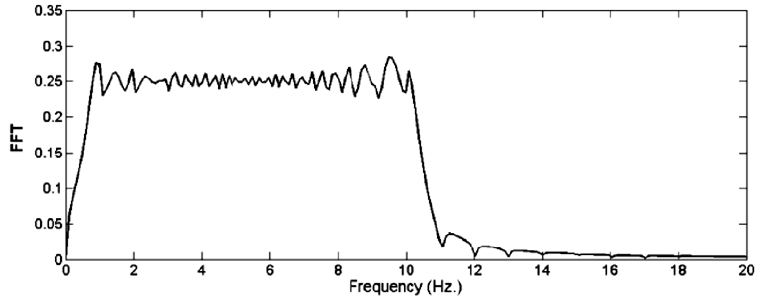
4. Input motion

The input base acceleration wave is a sinusoidal function with a frequency varying between 1-10 *Hz*. The amplitude of the input motion is kept constant at *0.05g*. Figure 4 illustrates the acceleration time histories and frequency content of the harmonic input motion. Equation (12) provides the relation for production of the acceleration time history illustrated in Figure 4 and adopted in sweep test to measure the natural frequency of the soil deposit.

$$\ddot{u}(t) = 0.5t\,\{sin[2\pi(t+1)t]\} \qquad 0 \le t \le 10 \qquad (12)$$



(a)



(b)

Figure 4. Input base motion; (a) time domain; (b) frequency domain (*FFT*).

5. Sweep test

A popular method to characterize site amplification has been the use of spectral ratios, introduced by Borcherdt [13]. The spectral ratio is calculated by taking the ratio of the Fourier amplitude spectrum (*FAS*) of a soil-site record to that of a reference-site record. The records should be from the same earthquake. The method is valid only if the distance between the two sites is much smaller than their epicentral distances (i.e. the source and path effects in the records are identical), and, therefore, the differences in the records are solely due to site effects. If $y(t)$ and $x(t)$ denote the discrete-time series of the recorded motions at a soil site and a nearby rock outcrop, respectively, the spectral ratio, $R(f)$, is calculated by the following equation (13):

$$R(f) = \frac{\sum_{j=1}^{2k+1} W(j)|Y(f-k-1+j)|}{\sum_{j=1}^{2k+1} W(j)|X(f-k-1+j)|} \tag{13}$$

where $X(f)$ and $Y(f)$ are the complex Fourier transforms of $x(t)$ and $y(t)$, respectively, $W(j)$ is the symmetric smoothing window with $2k+1$points, and f denotes the cyclic frequency. The smoothing aims to reduce the effects of noise that is always present in the records.

Sweep tests are carried out in physical modeling constructed on shaking table before applying the main shaking (Jamshidi et al. [15]). A wide range of frequencies are swept in order to find the natural frequency of the system. Autocorrelation (R_{xx}) and cross-correlation between input and response accelerations (R_{xy}) are first evaluated and the Fourier transform is employed to get the power spectral density of input (S_{xx}) and cross-spectral density between input and response (S_{xy}). The ratio of these at any given frequency (ω) is the transfer function ($G(j\omega)$) and is depicted against the frequency. Since it is always to use the constructed model for the main shaking tests, sweep tests are performed with very small acceleration input (0.5 m/s^2 in this study), which is far smaller than the real input acceleration. Thus the disturbance to subsoil before the main loading is guaranteed to be minimized. Frequencies between 1 and 10 *Hz* were swept at the rate of 1 *Hz/s*. The input accelerations at the base and the ground surface were calculated numerically and used to calculate the natural frequency of the system according to the procedure explained above.

The fundamental frequency of natural soil deposits in one-dimensional medium, $f_h^s$is inversely proportional to the depth of the soil deposit over bedrock and also directly proportional to the shear wave velocity as inferred from equation (14). Any change in shear wave velocity will move the alluvial deposits closer to or farther away from the condition of resonance and modify their amplification properties accordingly. Therefore, the shear wave velocity is a dynamic property of soils, crucially important for seismic analyses. It is related to the shear modulus, G, by the equation (15) [16]:

$$V_s \simeq 4Hf_h^s \tag{14}$$

$$V_s = \sqrt{\frac{G}{\rho}} \tag{15}$$

where ρ is the mass density of the soil.

Brad and Bouchun [17] showed that the natural frequency of two-dimensional alluvial deposits depends only on two parameters, namely, the one-dimensional natural frequency at the alluvial center and the shape ratio according to equation (16):

$$f_0 = f_h^s\sqrt{1 + (2.9H/0.5L)^2} \tag{16}$$

where f_0 is the natural frequency of *2D* natural soil deposit, *H* and *L* are the depth and length of alluvial deposit respectively.
The variability of the fundamental frequency of the model under study is found by conducting Monte Carlo sweep analyses which sweeps a frequency ranging from 1 to 10 *Hz* and numerically records the response at the control point representing the soil site.

6. Monte Carlo simulation

A Monte-Carlo simulation is a procedure, which seeks to simulate stochastic processes by random selection of input parameters in proportion to their joint probability density function. It is a powerful technique that is applicable to both linear and non-linear problems, but can require a large number of simulations to provide a reliable distribution of the response.
In what follows, the approach proposed in this paper, is employed to evaluate the effects of stochastic distribution of shear modulus representing a dynamic soil property on the resonance properties of natural alluvial deposits under seismic wave propagation. It uses the Monte Carlo simulation method in the sense that digital simulations of stochastic fields are combined with finite difference analyses.

The aforementioned approach referred to as "indirect simulation", although involving more computational effort, is deemed as more robust when compared to the "direct" approach - as the lack of data describing the cross-correlation structure of soil properties could lead to estimations with strong dependence on the selection of the empirical correlations with field test results. The Monte Carlo procedure followed in the present study involves three basic steps [18]:

1- Estimation of the statistics of spatial variability (spatial trends, spatially dependent variance, probability distribution functions and correlation structure) based on the results of extensive field measurement programs.

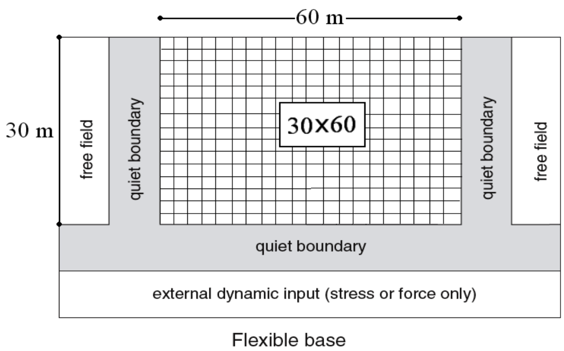
2- Digital generation of sample functions of a two-dimensional, non-Gaussian stochastic field, each simulated sample function representing a possible realization of soil property values over the analysis domain.

3- Deterministic finite difference analyses using stochastic parameter input derived from each sample field of soil properties; a sufficient number of finite difference simulations have to be performed to derive the statistics of the response.

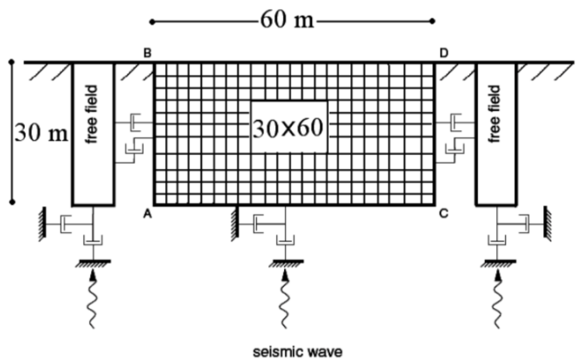
In such a technique, the governing equation is solved many times, each with a different set of the parameter of interest. Any set of values is an equally probable representation of this parameter. Finally, the results of the model are statistically analyzed to provide a stochastic output. In contrast, deterministic solutions define unique values for the results. Thus, stochastic solutions have the advantage of assessing uncertainty in model output due to parameter variability.

7. Dynamic modeling

Spatial variability of mechanical soil properties is modeled in two dimensional plane strain condition. Time and frequency-domain analyses were performed on the soil as nonlinear elastic material by adapting hysteretic damping behavior. The finite difference model used in the dynamic analysis is a statistically variable soil profile of 60 m horizontally and 30 m vertically as shown in Figure 5. It overlays a flexible bedrock where the input seismic motion is prescribed. The lateral extent of the soil is truncated by a transmitting boundary which eliminates spurious reflective waves and simulates the missing part of soil extending to infinity. For the problem under study, 1800 quadrilateral finite difference elements are used. The mesh was fixed at the base with quiet boundary, and restrained only vertically at the sides to model the free field conditions.



(a)



(b)

Figure 5. Dynamic model and boundaries; (a) quiet boundaries and (b) free field [19].

The lateral boundaries of the main grid are coupled to free-field columns through viscous dashpots similar to the quiet boundaries developed by Lysmer and Kuhlemeyer [20]. Along the free-field columns a *1D* calculation is carried out in parallel with the main grid calculation. In this way, if the main grid motion differs from that of the free-field, for instance due to waves radiating from the walls, then the dashpots are activated to absorb energy.

In static analyses, fixed or elastic boundaries (e.g., represented by boundary-element techniques) can be realistically placed at some distance from the region of interest. In dynamic problems, however, such boundary conditions cause the reflection of outward propagating waves back into the model and do not allow the necessary energy radiation. Therefore quiet boundaries must be applied at the model boundaries. In order to apply quiet boundary conditions along the same boundary as the dynamic input, the dynamic input must be applied as a stress boundary, because the effect of the quiet boundary will be nullified if the input is applied as an acceleration (or velocity) wave. The velocity record is converted into a shear stress wave using equation (17):

$$\sigma_s = 2(\rho C_s)v_s \tag{17}$$

where σ_s is applied shear stress, ρ is mass density, C_s is shear wave velocity, and v_s is input shear particle velocity.

The probability distribution of the natural frequency is studied by applying acceleration time histories at the surface midpoint of the finite difference mesh for different realizations of the same stationary random field. For the stochastic analyses, different *COV* of shear modulus are selected for comparison. The values of COV_G considered are 20, 40, 60, 80 and 100%. Properties of the soil in deterministic and stochastic phases are indicated in Table 1.

Table1. Material properties in stochastic analyses.

Properties	Symbol	Soft soil	Medium soil	Stiff soil
Density	ρ (kg/m^3)	1800	1900	2000
Poisson's ratio	v	0.35	0.35	0.35
Shear modulus	G (MPa)	32.4	46.3	74.1
Vertical correlation length	δ_V(m)	1	1	1
Correlation anisotropy	δ_H/δ_V	1, 20, 40	1, 20, 40	1, 20, 40
Coefficient of variation of shear modulus	COV_G(%)	0, 20, 40,60,80,100	0,20,40,60,80,100	0,20,40,60,80,100

It should be noted that laboratory tests such as resonant column or cyclic triaxial cell cannot reproduce the in-situ fabrics of the grains skeleton. Furthermore, the shear stiffness measured in the laboratory is not influenced by aging or preloading effects. These effects may lead to an increased in situ G_{max} [21]. Thus, laboratory tests are usually deemed to under-estimate the in-situ values of G_{max}. The shear modulus data presented in Table 1 are therefore increased according to a correlation diagram given in Figure 6. It shows that the ratio of small strain ("dynamic") and large-strain ("static") stiffness moduli is a function of the static values. The selected ranges for shear modulus in Table 1, fits with soil type *IV* according to the Iranian code of practice for seismic resistant design of buildings [22].

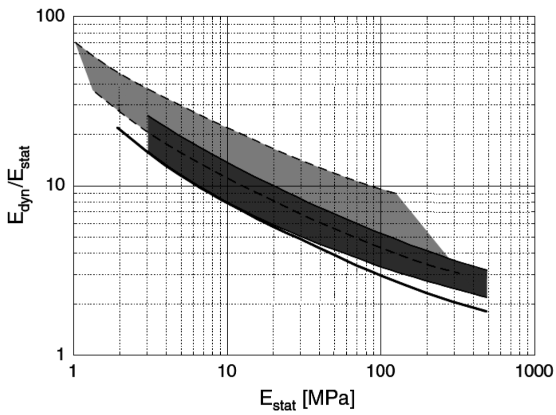


Figure 6. Comparison of the correlation E_{dyn}/E_{stat} by Alpan [21].

8. Random field modeling

The in-situ soil property variation is represented by the mean value, the coefficient of variation and the correlation distance, the most representative parameters for stochastic design. In present study, shear modulus, *G* is considered as a random variable and assumed to be a correlated log-normally distributed parameter introduced by mean, μ_G standard deviation, $\sigma_G$and different correlation distances, δ_V and δ_H.The use of lognormal distribution is appropriate in geotechnics as the soil properties are strictly non-negative and the distribution also has a simple relationship with normal distribution.

The total computation process is conducted by developing a 'FISH' code in FLAC *2D*. Monte Carlo simulation approach is used in generation of correlated *2D* realizations of random field. In present study, values of COV_G lie in the range of 20–100% (Table 1). Mean shear modulus, $\mu_G$bears the value of 32.4, 46.3 and 74.1 *MPa* and are kept constant throughout the domain. The study, indeed aims at investigating the effect of uncertainties embedded in shear modulus estimation on dynamic behavior of alluvial deposits. For each set of statistical parameters given in Table 1, Monte Carlo simulation is adopted by performing 500 realizations of shear modulus random field. The sufficiency of the number of stochastic analyses in Monte Carlo simulation scheme was examined by monitoring the mean natural frequency estimation error and *COV* of the natural frequency for the worst case, i.e. at COV_G=100% against the number of realization as illustrated in Figure 7.

Mean natural frequency estimation error defined by equation (18) refers to the normalized relative error between the mean results from truncated analyses and 500 realizations. This is used to establish the sufficient number of realization.

$$MNFEE = \frac{(\mu_{NF} - \mu_{NF_{500}})}{\mu_{NF_{500}}} \times 100 \tag{18}$$

where *MNFEE* is the mean natural frequency estimation error in percent; μ_{NF} is the mean natural frequency; and μ_{NF500} is the mean natural frequency when the number of realizations is 500.

Results indicate that compromising on 500 realizations would enable to capture the whole stochastic behavior for different input parameter sets and makes sense in both views of accuracy sought and the analysis cost which has direct relation to the number of realizations. This is endorsed by observation from Figure 7 that the mean natural frequency estimation error vanishes when the number of realizations approaches 500.

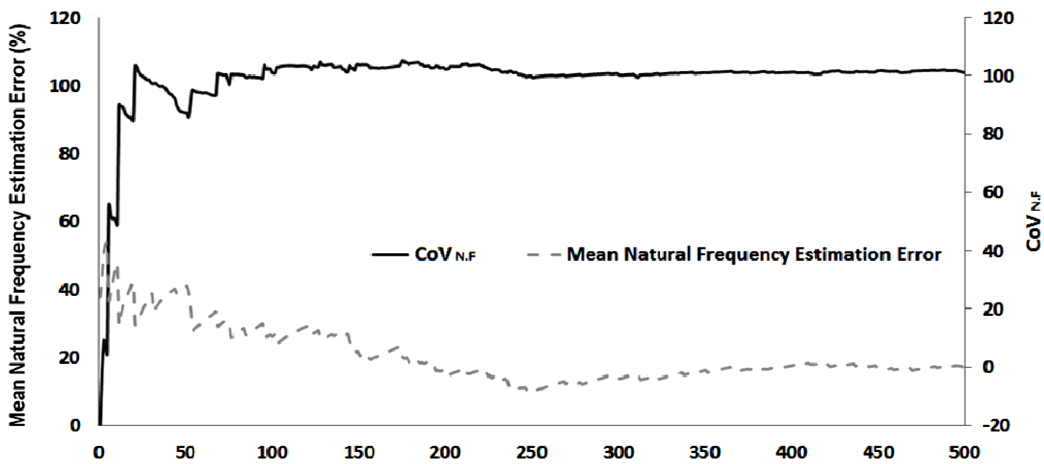


Figure 7. Sufficiency test for the number of realizations
(μ_G = 74.1 MPa, COV_G=100%, δ_V=1 m and $\frac{\delta_H}{\delta_V} = 1$).

Figure 8 illustrates a flowchart of the Monte Carlo simulation scheme by random finite difference modeling through repeated analyses. It describes the computational processes in a more tangible manner.

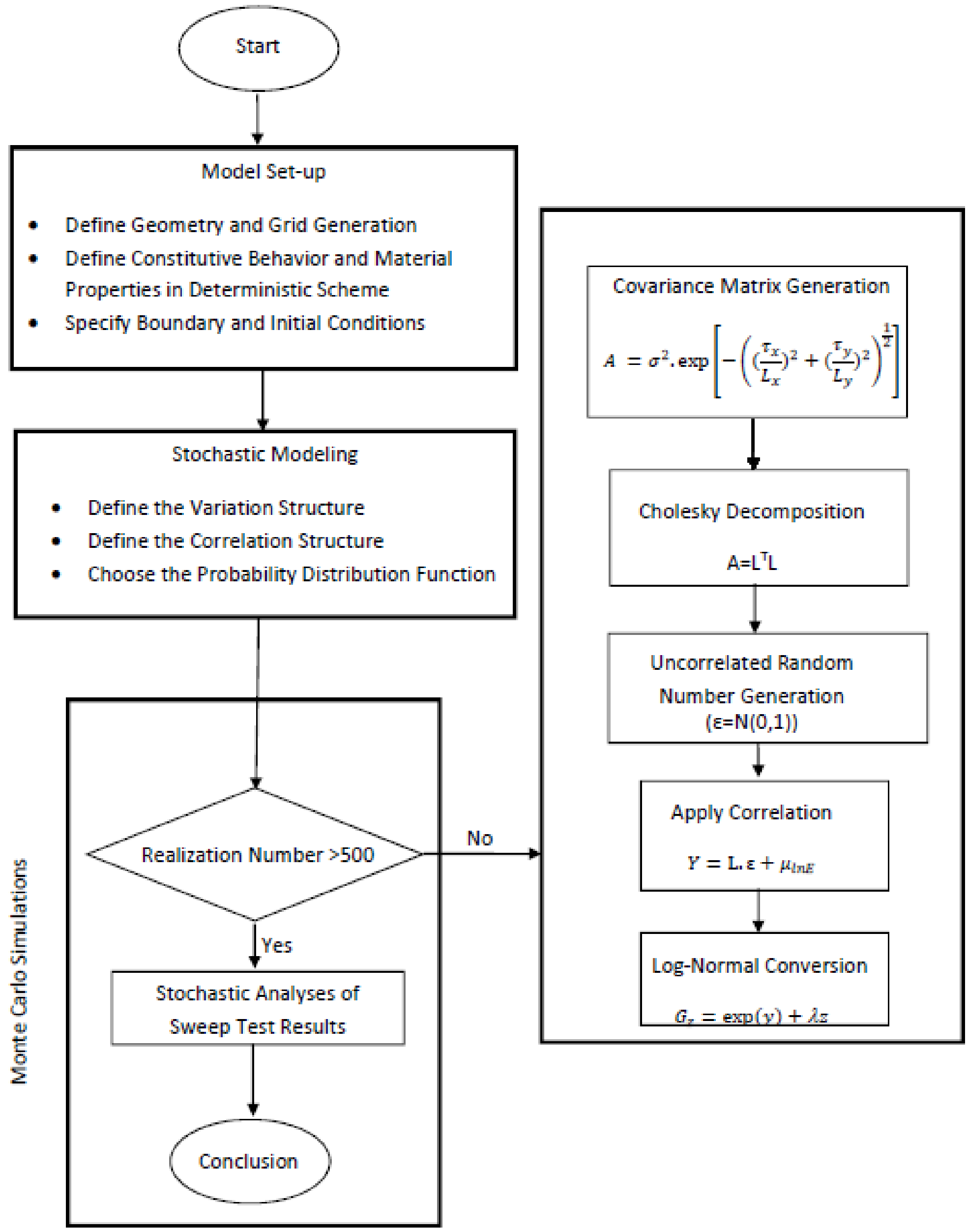


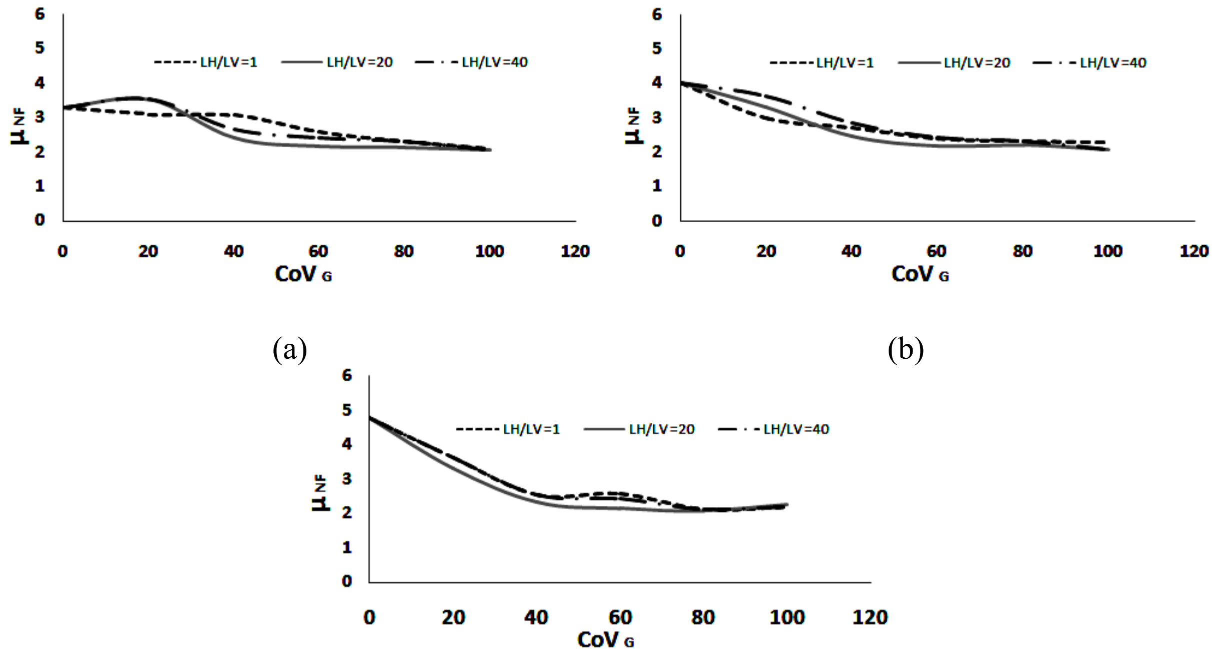
Figure 8. Flowchart of the computation process used in current study.

9. Results and discussion

This study considers the stochastic property of shear modulus, by investigating its effects on the extreme ground surface acceleration statistics (time domain), as well as on the mean transfer function (frequency domain). Also, the influence of the correlation lengths of shear modulus on output parameter statistics is investigated.

In-situ and laboratory determination of the coefficient of variation of shear modulus is troublesome and a large number of samples are tested very rarely. In order to cover a wide range of variability for shear modulus different coefficients of variation ranging from 0 to 100% were considered. Variability in shear modulus influences the natural frequency statistics for alluvial deposits. To this aim, the transfer function is calculated and plotted against frequency and the natural frequency statistics are obtained by extremizing the transfer functions for different realizations.

Results of stochastic analyses for different sets of statistical parameters for shear modulus introduced earlier are provided in Figure 9 in terms of mean natural frequency, μ_{NF} against the coefficient of variation of shear modulus, COV_G. Indeed the influence of the spatial variability of shear modulus on mean natural frequency of alluvial deposits bearing different soil conditions and anisotropic correlation structures is highlighted.



(a) (b) (c)

Figure 9. Mean natural frequency variation with the spatial variability of shear modulus; (a) soft clay; (b) medium clay; (c) stiff clay.

It is interesting to note from Figure 8, that the correlation distance of the shear modulus at horizontal direction has a little effect on the natural frequency of deposits while direction of shear wave propagation is vertical. This is as expected as the correlation length in vertical direction, δ_V is constant through analyses and correlation anisotropy implies different horizontal correlation lengths and this is believed to have minor effect when the wave propagation direction is vertical because each horizontal layer acts in average sense and variation in correlation length does not affect the overall averages. The abovementioned effect is justified by the fact that the lateral boundaries prevent radiation of refracted and reflected waves through energy absorbing elements. Another observation from Figure 9 is that the mean natural frequency decreases with the COV_G increasing. This behavior can be explained better when depicting the variation of natural frequency along with their spectral ratios in Figure 10 for different soil conditions and the degrees of variability of the shear modulus.

Figure 10 illustrates the Fourier amplitude against the frequency or the frequency contents of different realizations. The sparsely distributed points in the chart are obtained from calculation of the dominant natural frequencies of different stochastic models substantiated by different realizations employing sweep test analyses explained earlier. Each point corresponds to a single analysis and represents the dominant natural frequency of that single realization. The goal of this illustration is to compare the dominant frequency of the stochastic varying models with a representative homogeneous condition while preserving the overall mean shear modulus constant throughout the domain of analysis. Indeed each realization has multiple modal frequencies, but this study is to reflecting the dominant natural frequency of heterogeneous models by allocating them through a benchmark and representative homogeneous model specified by a few distinctive natural frequencies.

Superimposed on the same graph is the theoretically calculated transfer function for the representative homogeneous model in solid line extremizing at modal frequencies. The mean shear moduli were adopted in equation (19) to calculate the theoretical transfer function assuming zero damping.

$$|F(\omega,\xi=0)| = \frac{1}{\sqrt{cos^2\left(\frac{\omega H}{V_{ss}}\right) + a_z^2 sin^2\left(\frac{\omega H}{V_{ss}}\right)}} \tag{19}$$

where F is the transfer function, $a_z = \frac{\rho_s V_{ss}}{\rho_r V_{sr}}$ is the impedance ratio, $\rho_s$is the mass density of soil layer, $\rho_r$is the mass density of elastic bedrock, V_{ss} is the shear wave velocity of soil, V_{sr} is the shear wave velocity of bedrock and ω is the circular frequency.

According to the results illustrated in Figure 10, it is obvious that for low *COVs* of inherent variability of shear modulus, points corresponding to the dominant natural frequency of stochastic models are nicely fitted and nested to the places assigned to the modal frequencies of representative homogenous model. This obviously means that for low coefficients of variability of shear modulus the natural frequency of soil stratum is estimated with less uncertainty and their behavior is similar to the deterministic mode. Diffusion and scarcity of the points are clearly observed when the *COV* of shear modulus gets remarkable. This implies the inadequacy of deterministic analyses for highly variable stochastic system. Natural alluvial deposits bearing inherent variation of the deformation and strength parameters have been widely pointed out in literature to be inherently variable. Phoon and Kulhawy [10] can be conferred for more insight into the problem.

Another observation from Figure 10 is that the overall concentration of the natural frequency points moves toward left when the coefficient of variation of shear modulus increases. This implies that the mean natural frequency decreases with the increase of COV_G and it was also confirmed the same in Figure 9. This behavior can be attributed to the occurrence of soft clusters inside the discretized model. They will probably cause significant reflection and refraction leading to the conclusion that such soft points dominate the dynamic behavior of the system and the decreasing in mean natural frequency of the deposit is then expected.

Sakayama and Tonouchi [23] presented a logging data in which a soft thin deposit inside an interface caused a significant wave reflection. However, Towhata [4] believes that generally, such a thin layer does not seem to extend on a straight line and to be wide enough to affect the regional earthquake response of the ground.

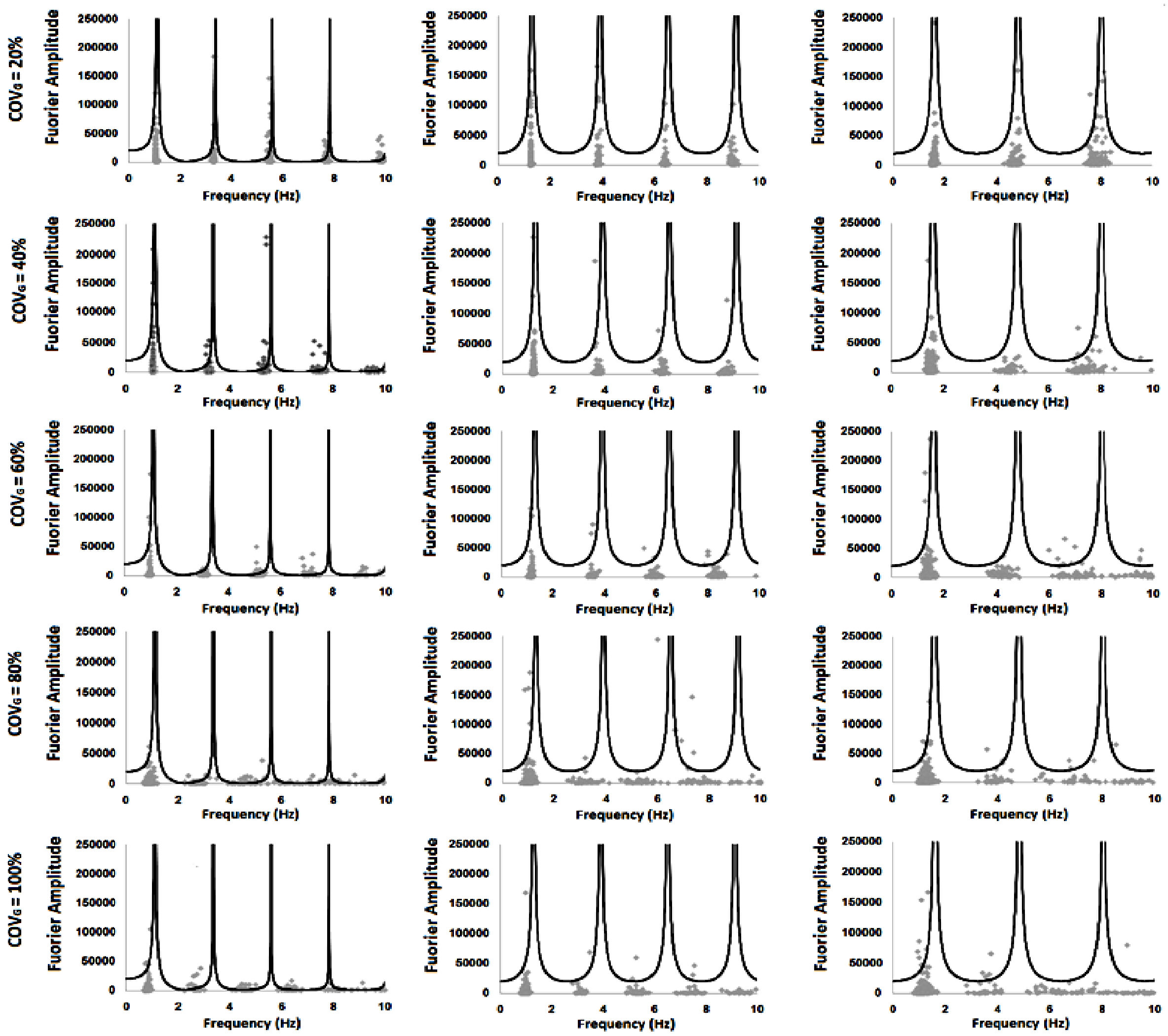


Figure 10. Results of natural frequencies for stochastic analyses superimposed by deterministic curves.

For COV_G more than 60% the uncertainty in natural frequency estimation becomes more highlighted and the frequency data is observed to be sparser in comparison to low variability cases. This means that the coefficient of variation of natural frequency increases as the COV_G increases. This observation is confirmed by calculating and depicting the coefficient of variation of natural frequency against the COV_G in Figure 11 for different soil conditions.

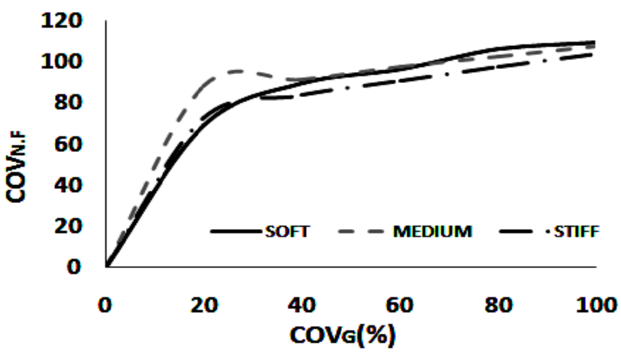


Figure 11. Coefficient of variation of natural frequency against the COV_G for different soil types (δ_V=1 m and $\frac{\delta_H}{\delta_V} = 1$).

9.1. Theoretical interpretation

A very simple theoretical model for uniform soil on rigid rock illustrates that the frequencies at which strong amplification occurs depend on the geometry (thickness) and material properties (*S*-wave velocity) of the soil layer. For many naturally occurred soil profiles, the distribution of shear-wave velocity with depth does not fit any of the cases for which closed form solutions are available, and, therefore, numerical methods are usually sought to compute the fundamental frequency. Figure 12 illustrates how the shear modulus is spatially variable through the depth of natural alluvial deposits. The realization represents a deterministic distribution of shear modulus. It is indeed considered to be a *n*-layer profile while *n* can be a large number. In order to clarify the importance of weak zones in response behavior of natural alluvial deposits, a simplified procedure developed by Dobry et al. [3] was employed for estimation of the fundamental frequency of two-layer soil profile.

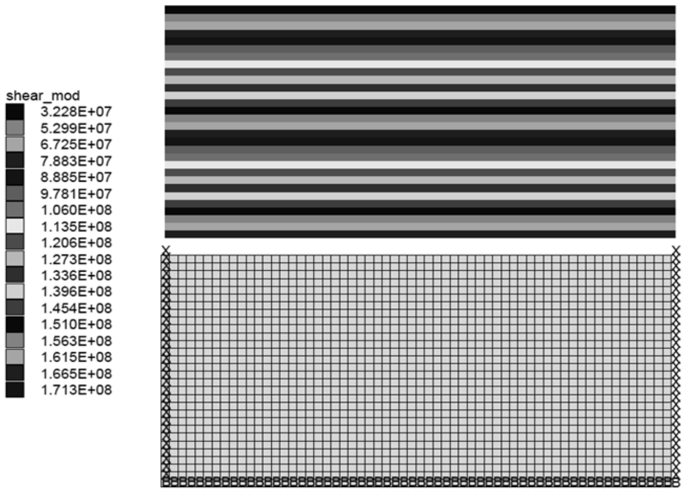


Figure 12. *1-D* deterministic realization of spatially variable soil profile.

Two soil profiles were chosen to show the effect of the coefficient of variation of shear modulus on fundamental frequency of deposits theoretically. Simple chart provided in Figure 13 is utilized to show the mentioned effect. The length and density of both profiles are kept constant at *2 m* and *2000* kg/m^3 respectively. The first soil profile consists of two different shear moduli equal to $G_1= 34$ *MPa* ($V_{s1}=130$ *m/s*) and $G_2= 46$ *MPa* ($V_{s2}=151.5$ *m/s*), and the second soil profile, is formed of two different shear moduli equal to $G_1= 23$ *MPa* ($V_{s1}=107$ *m/s*) and $G_2= 57$ *MPa* ($V_{s2}=168$ *m/s*) while maintaining the average shear modulus constant at *G= 40 MPa*.

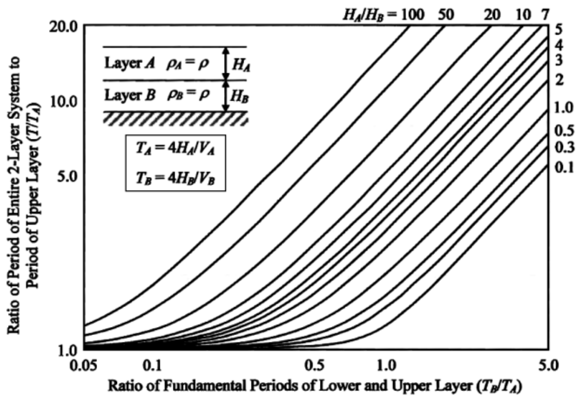


Figure 13. Fundamental period of two-layer system [3].

According to the aforesaid theoretical procedure provided in Figure 13, the calculations imply that the first profile with lower coefficient of variation of shear modulus shows higher natural frequency than the second profile. This implies that introducing weak zones in spatial variability realizations while maintaining a constant overall mean shear modulus value, is expected to cause a decrease in profile natural frequency.

This theoretical interpretation is inconformity with finding of Monte Carlo analyses illustrated in Figure 14 for different soil conditions. This means that the increase in COV_G introduces more variability in shear modulus which itself implies that more probability of the formation of weak zones is expected.

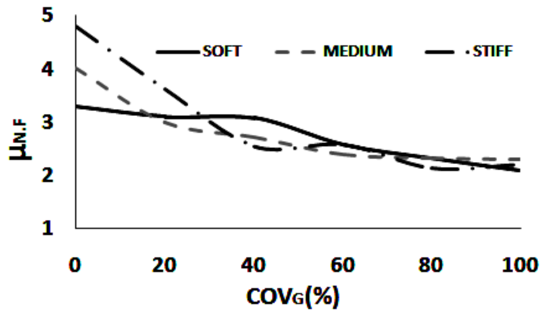


Figure 14. Mean natural frequency variation with COV_G for different soil types (δ_V=1 m and $\frac{\delta_H}{\delta_V} = 1$).

10. Conclusions

The major contribution of the present study is to investigate the effect of uncertainty in soil shear modulus estimation on resonance behavior of naturally occurred alluvial deposits. It is observed that there is a significant variation in natural frequency due to the variation of shear modulus. Variation of natural frequency of soil deposits is stated in term of mean natural frequency and the coefficient of variation of natural frequency. From the results of numerical dynamic sweep test analyses it is concluded that:

1- The mean natural frequency of spatially variable natural alluvial deposits decreases due to an increase in variability of shear modulus or the shear wave velocity in other words. This behavior is justified by formation of weak zones introduced by high variability in natural deposits. The results of the current study emphasizes that ignoring the spatial variation of shear modulus of natural alluvial deposits will in effect lead to overestimation of the natural frequency of soil deposits.
2- The coefficient of variation of natural alluvial deposits increases with the increase in variability of the shear modulus. This means that the uncertainty involved in natural frequency estimation increases with the increase in uncertainty embedded in shear modulus or shear wave velocity estimation. Another implication is that the reliability of conventional deterministic analyses fades when the soil deposit is highly variable spatially.
3- Monte Carlo simulation technique combined with numerical analysis is a very useful tool to explore and analyze the stochastic behavior of the natural alluviums.

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